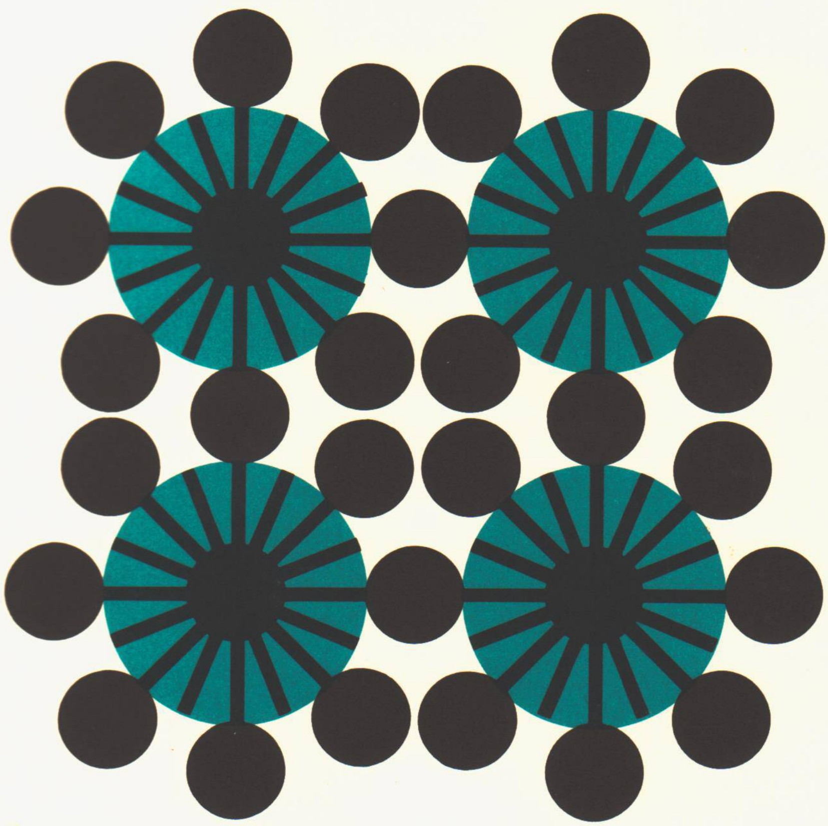
EXPERIMENTS WITH CALCULO Analog Computer Kit



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THE SCIENTIST'S WAY OF LEARNING

Experiments With
CALCULO
Analog Computer

by

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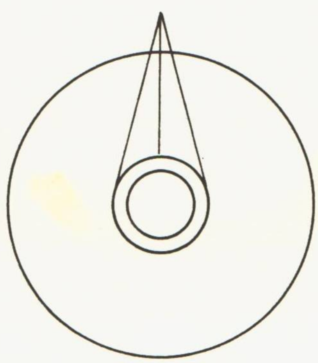

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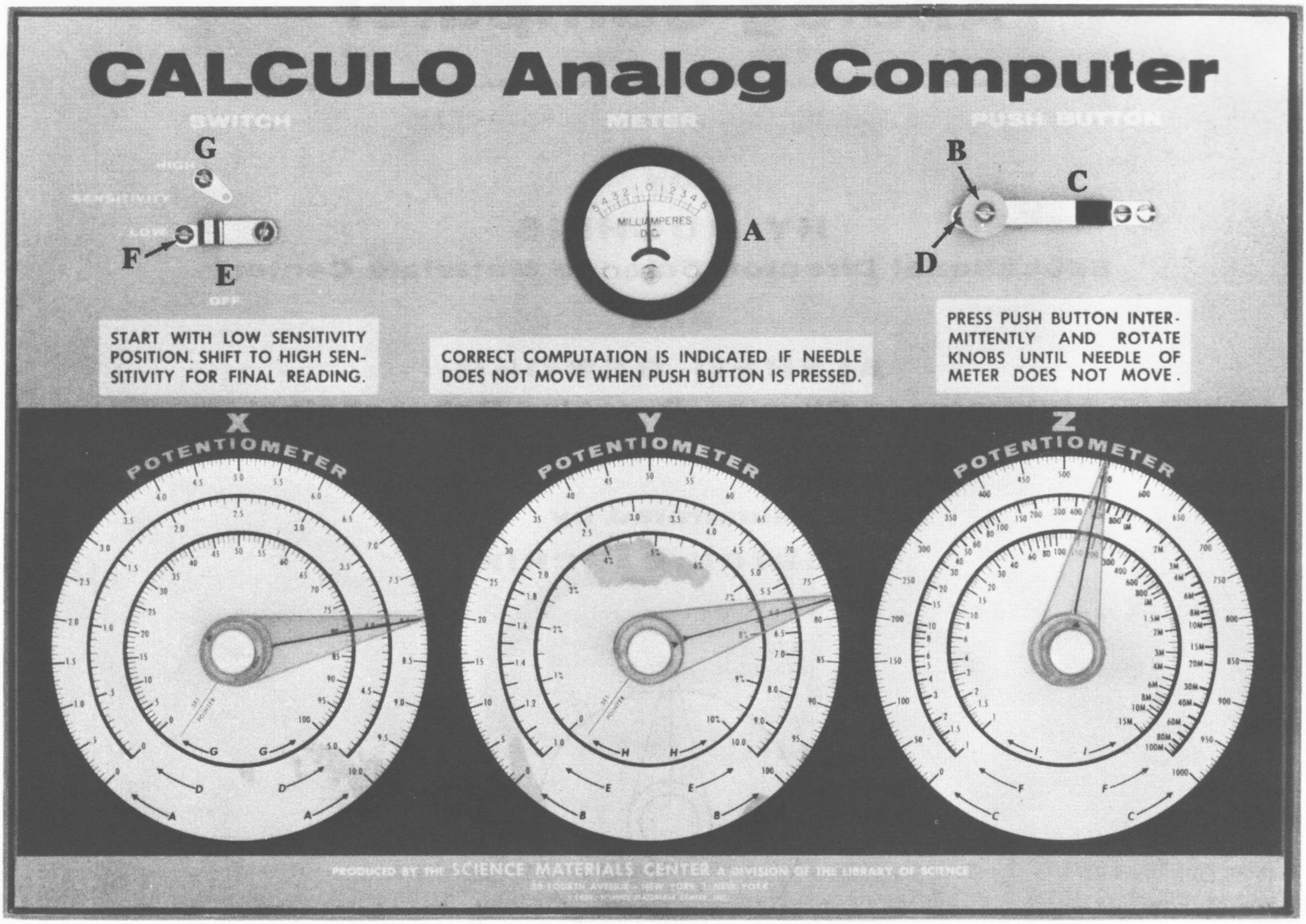


Fig. 1—Appearance of the front of your Calculo Analog Computer when finished.

THE AGE OF COMPUTERS

A new age of industry and science is dawning. Automatic machinery is rapidly replacing the old-fashioned hand-operated machine. Assembly lines are rapidly shifting to automatic operation. Bookkeeping and record-keeping are more and more often performed by computers of many types. Airplanes can glide into runways with automatic guidance systems. Automobiles may some day be operated by automatic steering systems.

A rapidly increasing number of scientists, mathematicians, and technicians are needed to design and operate these new mechanical and electrical "brains." Your Calculo Analog Computer kit provides the first step in understanding the principles upon which many computers operate. It can provide an important start for young people preparing for a profession that involves knowledge of computers. It will prove useful to technicians and engineers who wish to know more about computers.

There are two basic types of computers, digital and analog. Both operate electrically. The digital computers add, subtract, multiply, and divide by making use of actual digits. Analog computers, on the other hand, operate on the basis of electrical analogues (analogies or resemblances). Analog computers may be designed to handle many types of complex mathematical formulas. In effect they resemble electrical "graph" machines which show the way in which a change in one factor affects others. They are particularly useful in solving problems associated with airplane design, engines, electronic parts, radio antennas, hydraulic devices, and guidance systems.

Analog computers are not so accurate as digital computers in specific calculations. The latter can compute accurately to 10 or more digits. But when the effect of one changing quantity upon another is being calculated, digital computers can provide a table of values only for certain specific solutions. They do not give a continuous picture of the change that occurs in one quantity as one or more other quantities change, as analog computers do.

Analog Computers are limited in accuracy. Errors of 1% (1 part in 100) are considered good, even in fairly expensive computers. Improvements in accuracy can be achieved only with great expense, with a limit reached at about .01% (1 part in 10,000).

Your low-cost Calculo Analog Computer is designed for an accuracy in multiplication and division of about 5%. By means of certain tricks (explained later), you may be able to increase accuracy somewhat. The purpose of your computer is not to serve as an accurate computing device, but rather as an estimating machine and, more important, as a basic introduction to the general method of operating analog computers.

ASSEMBLING THE COMPUTER

You will need the following tools: pliers or small wrench, knife with a sharp point, ruler, fountain pen, steel wool, medium screwdriver, very small screwdriver, and wire stripper or cutting pliers for removing insulation. You will also need two flashlight cells, size D. (These have not been included in the kit because they deteriorate in storage.)

1) Remove the inside printed computer box from its outer box. The appearance of the front of this inside box with all parts assembled is shown in Fig. 1.

2) Empty the contents of the Parts boxes into a large dish. There are two different sizes of bolts. Separate the larger (#6) bolts from the smaller ones (#2). Identify the two sizes of nuts that fit these bolts. Place all nuts and bolts of the same size together. In the instructions that follow, we shall refer to the larger bolts and nuts as #6 and the smaller ones as #2.

3) In Fig. 2, note the positions of two flashlight-cell boxes (A and B). Note that the end marked (+) is closer to the upper part of the computer box while the end marked (−) is closer to the bottom.

Attach connecting bolts, washers, and nuts to these cell boxes as shown in Fig. 3.

Insert a #6 bolt (A in Figure 3) and flat washer (B) at one end of the box from the inside. On the bolt outside the box, add a regular washer (C), a lock washer (D), and nut (E). Tighten firmly with a wrench or pliers while using a screwdriver to keep the bolt from turning. Do the same at the other end of the box (F). Insert a flashlight cell into the box with the top and bottom of the cell in the exact position indicated by the drawing on the outside of the box. This position is important. If it is reversed, electrical connections will be incorrect and current may not flow. Check to see that the top and bottom

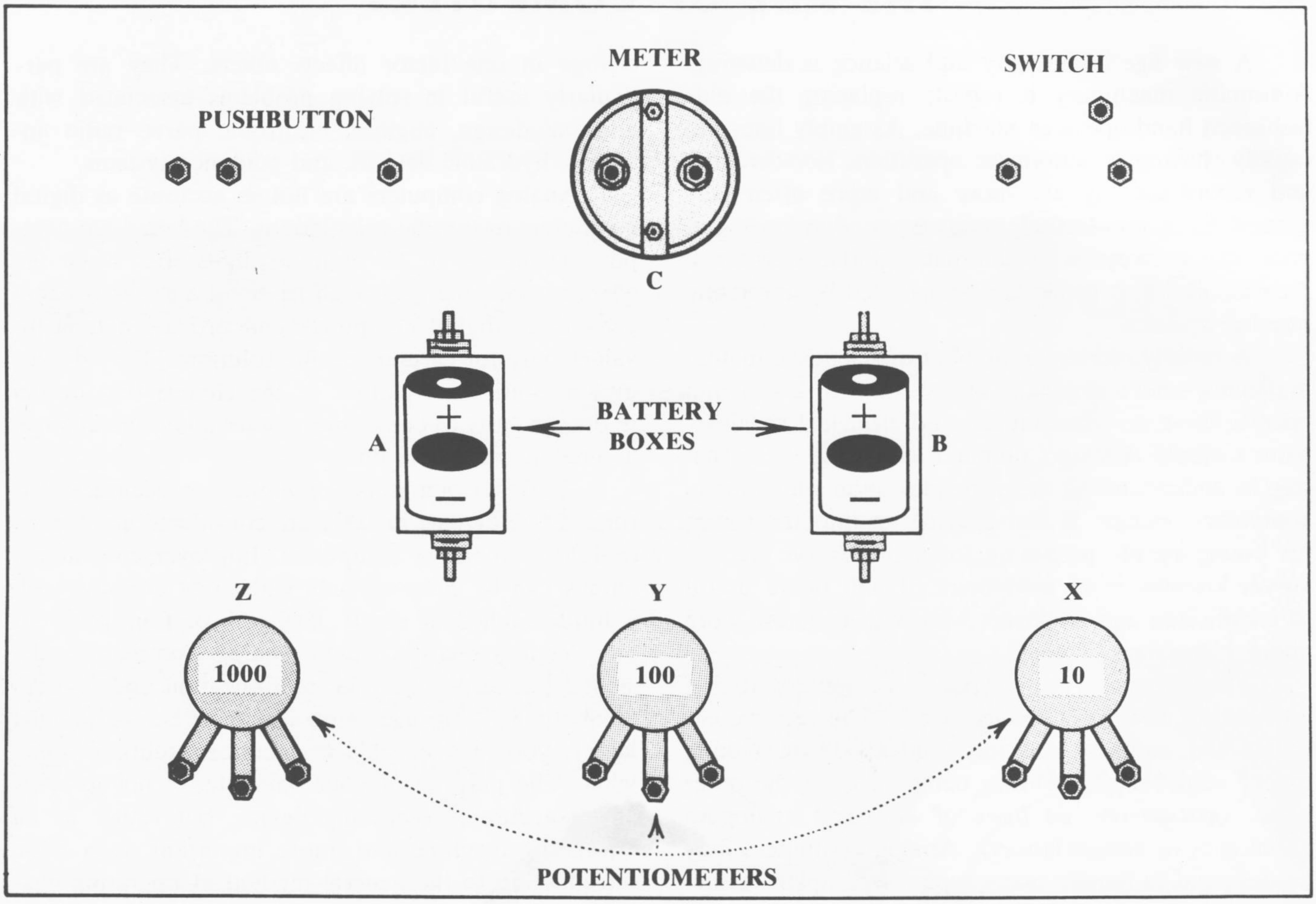


Fig. 2—Appearance of the back of the computer with all parts in place, before wiring.

of the cell make firm contact with the heads of the bolts. Should there be any indication of a loose fit, the head of the bolt should be moved inward by adding an extra washer under the head of the bolt at one or both ends. Remove the flashlight cell.

4) There is a strip of special two-sided gummed tape in your kit. Cut off two pieces equal in length to the length of the cell boxes. Note that both sides of the tape have a slit running down the center along the length. This slit enables you to remove the protective covering from the tape. Peel off the protective layer on one side, and place the adhesive side of the tape against the side of the cell box that is labeled "Attach This Side to Surface," as shown in Fig. 4.

Now peel off the protective coating from the exposed side. Place the computer box face downward on a large, *clean*, flat surface on a table. Press the adhesive side of the flashlight cell box down against the box, as shown in position A of Fig. 2. Reach into the box and press all parts of the bottom of the cell box against the computer box to ensure good contact. The cell box is now firmly attached.

Replace the flashlight cell, making sure that the (+) terminal (top of cell) is touching the bolt marked (+). Lock the box with the special small flap on the side. Attach the second cell box in the same way, as shown at B in Fig. 2.

The placement of these boxes in the positions shown in Fig. 2 serves an important purpose. They support the center of the computer box and make it more rigid.

5) Remove the meter from the box. There is a U-shaped bracket (C in Fig. 2) on the back of the meter. Remove the U-shaped bracket from the meter by loosening the two nuts. Put the nuts aside in a place separate from the other parts in the kit. From the front of the computer, insert the meter into the hole in the box in the position shown at A in Fig. 1. Be sure that the meter is positioned with the needle pointing toward the top of the box. From the back, slip the U-shaped bracket over the two bolts of the meter, as shown at C in Fig. 2.

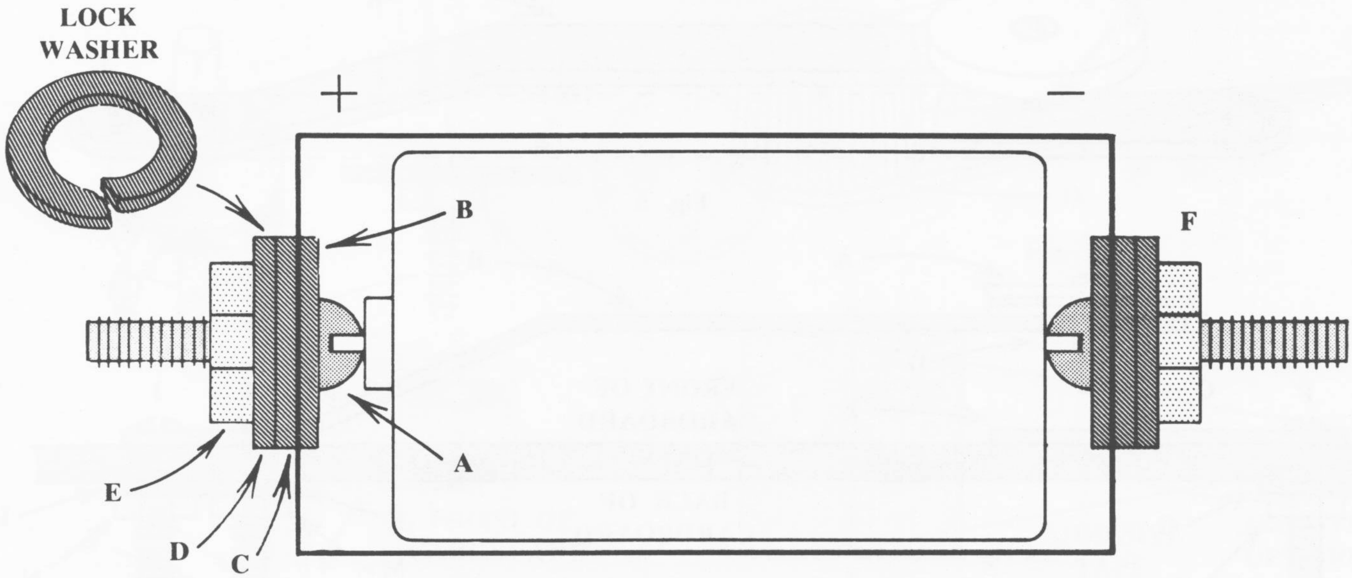


Fig. 3

Replace the washers and nuts on the two bracket bolts and tighten them firmly. As you tighten the nuts, the ends of the bracket push against the back of the computer board while the wide rim of the front of the meter is pulled against the board on the opposite side. As a result the meter is locked tightly to the board.

6) Locate the white push button disk (B in Fig. 1). Attach the disk (A in Figs. 5 and 6) to the push button strip (B in Figs. 5 and 6), using a #2 bolt (C in Fig. 6) and its nut (D). Tighten firmly with screwdriver and pliers or wrench.

7) The part that you have just made is to be attached to the push button section of the computer box as shown at C in Fig. 1. But first a contact strip must be attached, as shown at D in Fig. 1. The entire assembly of the push button is shown in Figs. 5 and 6. Note the flat brass strip at E in Fig. 5. Locate this strip among your parts. Place it on the part of the computer box as shown at D in Fig. 1. Insert a #6 bolt through the hole in the strip and the hole in computer box. Add a lock washer (G in Fig. 6) and nut (H) to the bolt (F) at the back of the computer box. Be sure that the strip lines up properly, as shown at E in Fig. 5. Then tighten the nut firmly.

Now attach the push button (B in Figs. 5 and 6) to the board using two bolts (I and J), lock washers (K and L), and two nuts (M and N). Tighten both firmly.

Check to see that there is an open gap (O in Fig. 6) between the bolt (C) of the push button and the strip (E). If not, you will have to bend strip B slightly. A gap of about ⅛″ is desirable.

8) Set up the switch, as shown at E, F, and G in Fig. 1. Another view is shown in Fig. 7. Use one flat washer (A in Fig. 7) above the switch arm (B). Use 3 washers (C, D, E) between the switch arm (B) and the cardboard platform (F). Fasten all in place with a #6 bolt (G), lock washer (H) below the board, and a nut (I). Do not tighten too much, just enough so that the switch arm can rotate without being loose.

There are two oval-shaped brass strips in your kit, as shown at B and C in Fig. 8. Use a #6 bolt to attach each strip to the computer box, as shown at F and G in Fig. 1. Be sure that the strips line up pointing toward A in Fig. 8. Fasten each into place with a nut. Lock washers are not needed because the sharp points of the irregular hole in each strip act to grip the head of the bolt. Tighten firmly.

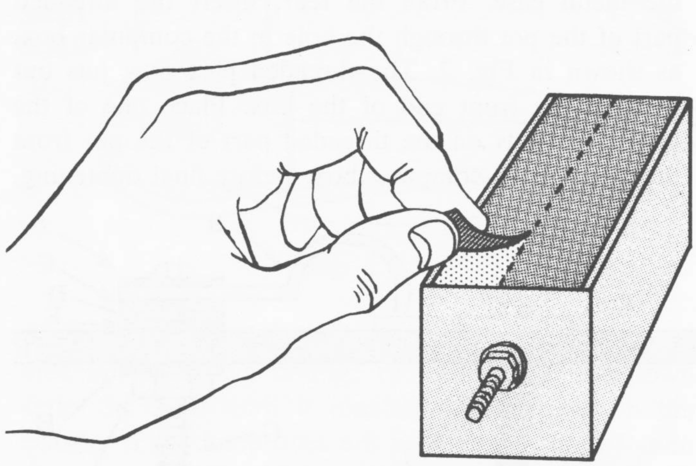


Fig. 4

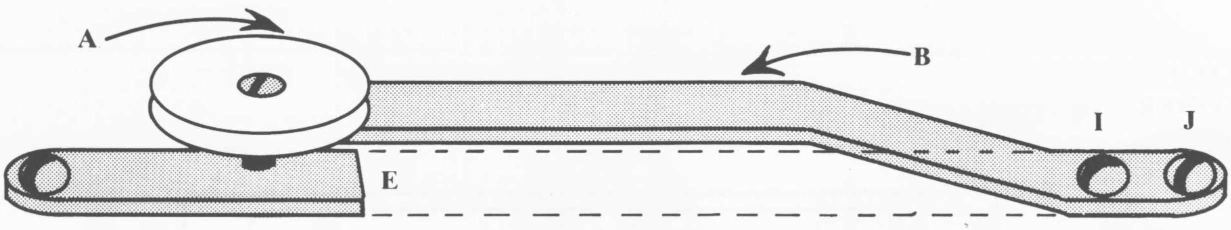


Fig. 5

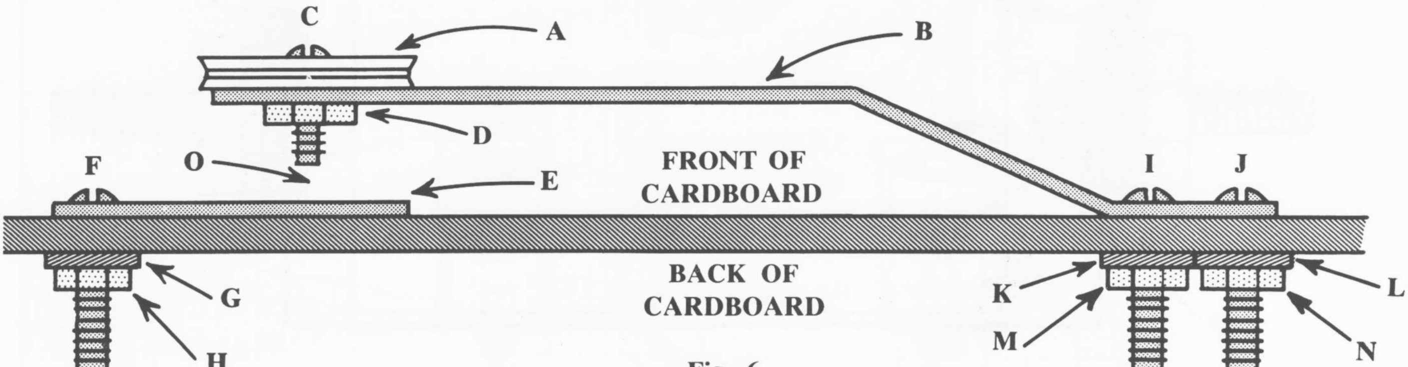


Fig. 6

9) Refer to Fig. 9. Fasten a #2 bolt (A) and nut (B) to the three strips of the three potentiometers, as shown in Fig. 9. Tighten each very firmly.

(Note: Henceforth we shall use the abbreviated designation *pot* instead of potentiometer.)

There are 6 very large nuts in your kit. Two are used to attach each pot to the computer box. Twirl a large nut about half way onto the threaded part of each pot, as shown at C in Fig. 10.

Examine Fig. 2, which shows the computer box from the rear. Note that pot Z (1000 ohms) is at the left, pot Y (100 ohms) is at the center, and pot X (10 ohms) is at the right. (When you turn the box around and view it from the front, the positions of the pots will appear in the proper order: X, Y, Z, as in Fig. 1.)

Select the pot marked with the number 1000 on the metal case. From the rear, insert the threaded part of the pot through the hole in the computer box, as shown in Fig. 2. The threaded part now juts out through the front end of the box. Place one of the very large nuts on the threaded part of the pot from the front of the computer box. Before final tightening, check the position of the pot from the back and see that its three terminals point downward, as shown in Fig. 2. Also check to see that none of the threaded part sticks up beyond the nut at the front. Note in Fig. 10 that the end of the nut (A) is flush with the bushing (B). If it projects out, remove the pot and rotate nut C so as to increase distance D somewhat. Insert once again and tighten with a nut on the front so that the threaded part does not project beyond the nut.

Do the same thing with the 100 ohm pot at the center and 10 ohm pot at the right (as seen from the back of the computer box).

10) The knobs (A in Fig. 11) and pointers (B) are now prepared. Examine a pointer and note that there is a groove (C) along the center. Place the pointer on a flat piece of cardboard or scrap wood with the groove on the upper side (A in Fig.

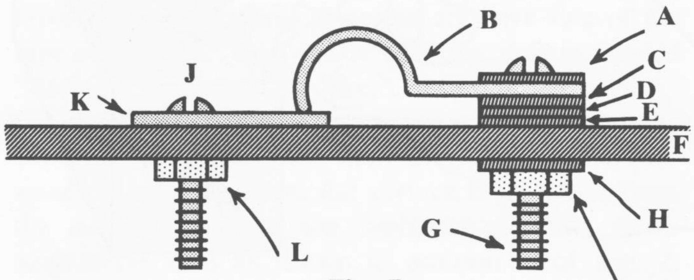


Fig. 7

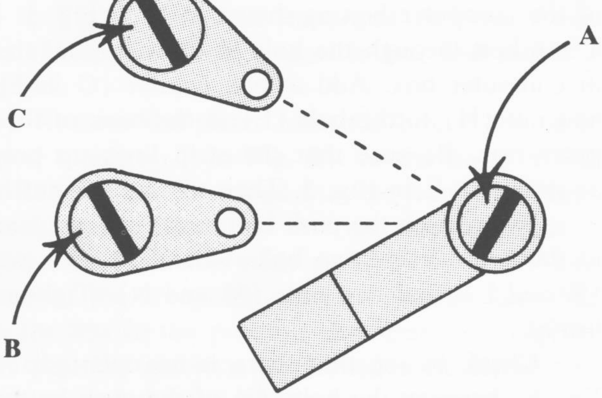


Fig. 8

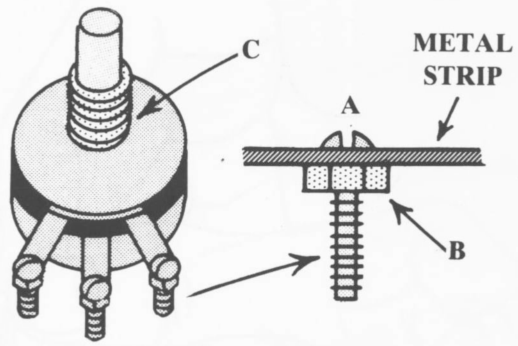


Fig. 9

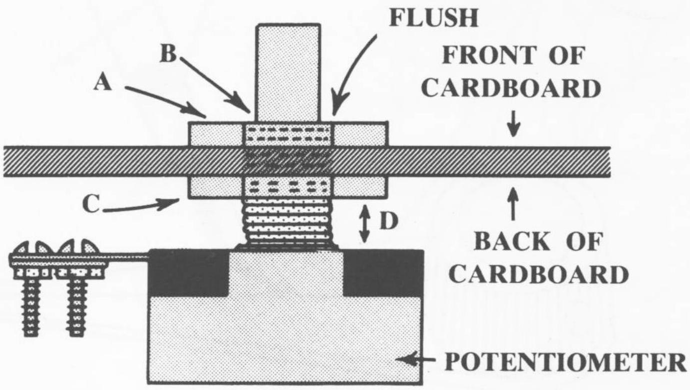


Fig. 10

12). In order to avoid scratching the table top, work on a thick cardboard or piece of wood.

Cut off about 1″ (or slightly more) of the two-sided gummed tape used for the cell boxes in step 4. Peel off the protective covering on one side, and place the adhesive side against the wide end of the pointer, as shown in Fig. 12. The edge of the tape at B should just barely overlap the round end of the pointer (C). An excessive overlap may leave too little for proper coverage at the opposite end (D). Be sure that there is also some overlap at points E and F.

With a sharp knife cut through the tape over the center hole in the pointer and cut out all the tape over the hole. Do this carefully and make the hole fairly clean. Now use scissors to trim off all the tape that juts out beyond the curved edge of the plastic pointer.

Carefully peel off the protective layer from the gummed tape to expose the adhesive on the upper surface. Do not wrinkle the tape, or if you do, lift the edge and reset.

Place the computer box face upward with the shafts of the pots upward. Place the pointer on the

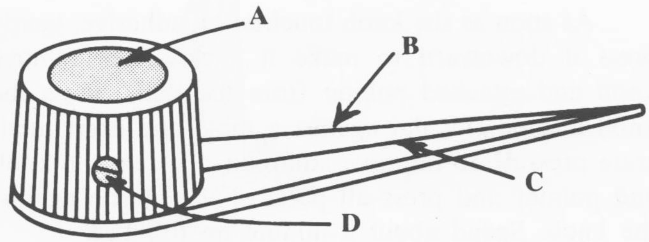


Fig. 11

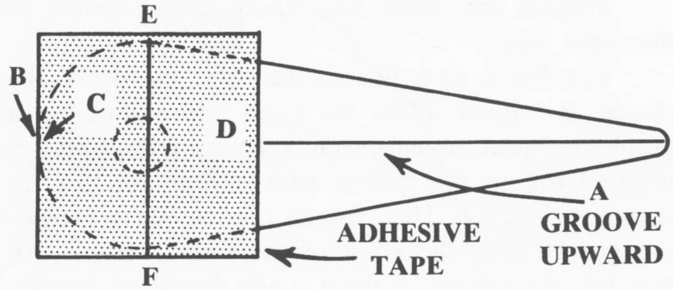


Fig. 12

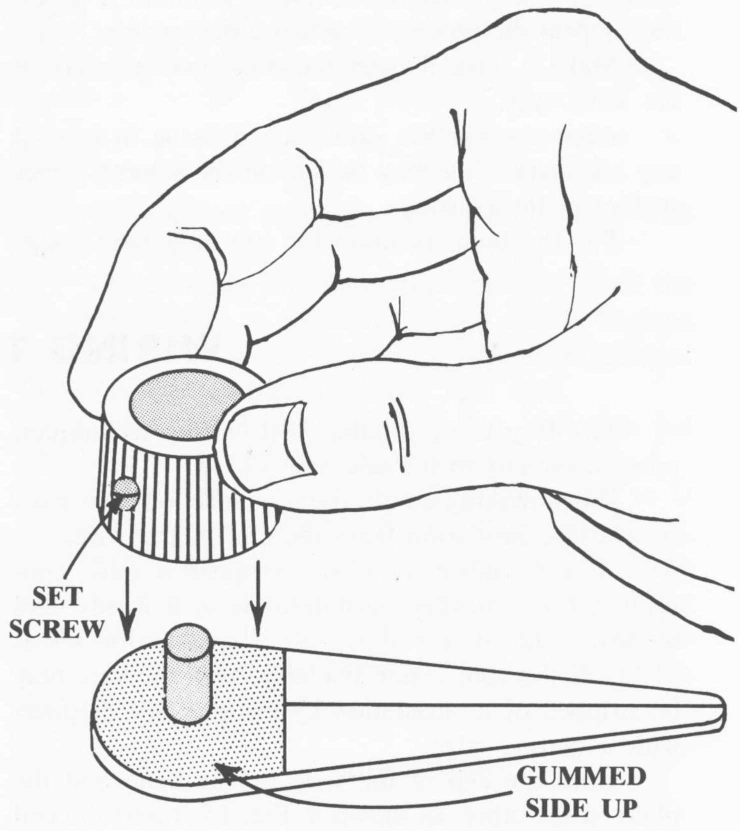


Fig. 13

shaft of any of the pots with the exposed adhesive side up, as shown in Fig. 13. Slide the knob down onto the shaft until it touches the adhesive on the plastic. If the knob does not fit onto the shaft, loosen the setscrew (Fig. 13) with a small screwdriver until the knob slides freely onto the shaft.

As soon as the knob touches the adhesive, gently press it downward to make it stick. Then remove knob and attached pointer from the shaft. Press the knob and pointer flat against a table and exert moderate pressure to improve adhesion. Pick up the knob and pointer and press all parts of the plastic against the knob. Spend about a minute on this task.

Now use a sharp knife to trim off any exposed gummed tape.

Prepare the other two knobs and pointers in the same way.

11) Put a dark line in the outer groove of the plastic as follows. Place the knob and pointer on a protective sheet of paper on a level table. Apply a drop of ink to the groove with a fountain pen, as shown in Fig. 14. Then spread the ink along the entire groove with the point of the pen. The ink will flow into the groove. If there is any excess ink outside the groove, blot it up with a blotter or a piece of tissue. Let the groove dry in a level position. If necessary, repeat the process to obtain a darker line.

Make a dark line on the other two pointers in the same way.

After the ink has dried, use a tissue to rub off any ink marks that may be left on the smooth upper surface of the plastic.

Put the pointers aside for use at a later stage.

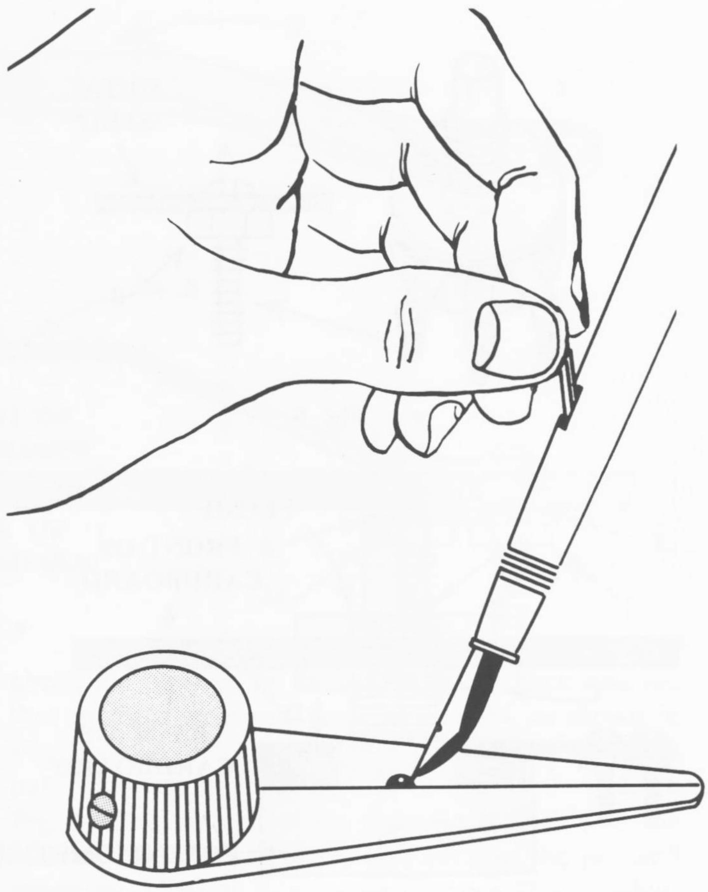


Fig. 14

WIRING THE CIRCUIT

12) If you are familiar with wiring techniques, proceed directly to the next step (13).

When making an electrical connection, you must remove the insulation from the end of the wire.

A tool known as a wire stripper is most convenient for removing insulation. It is a handy tool to have, and simple types are available for about $1.00. If this tool is not available, then the wire may be stripped of its insulation by using a pair of pliers with a cutting edge.

Hold the end of the wire in one hand and the pliers in the other, as shown in Fig. 15. Insert the end of the wire into the cutting edge of the pliers at A. Hold the wire with one hand (B). The thumb of the other hand presses outward against the edge of the handle at C. The fingers pull the handle of the pliers inward at D and E. The other fingers push outward at F and G. The combined outward and inward pulls may then be exerted to control the cutting action to any desired degree. The insulation must be cut but not too deeply, or the wire will also be cut through. Squeeze the pliers gently in order to cut the insulation but not the metal. Then pull one hand in the direction H and the other in the direction I. If the insulation has been cut through, it will part at point A and slide off at J. The bare metal will be exposed.

A bare wire section about 3/4" to 1" long will be sufficient for making a connection.

When cutting the lengths of wire required in the steps below, stretch the wire between the two points to be connected and then allow at least 2 extra inches, for the removal of insulation at each end.

When making connections, hook the wire around a bolt in a clockwise direction, as shown in Figure 16. Cross the wire as shown, and crimp it with pliers. Then, when a nut is tightened down on it, it will have little tendency to work itself loose. If the wire is twisted around the bolt in the opposite direction, it will tend to loosen as the nut is tightened.

The connections for the computer are shown

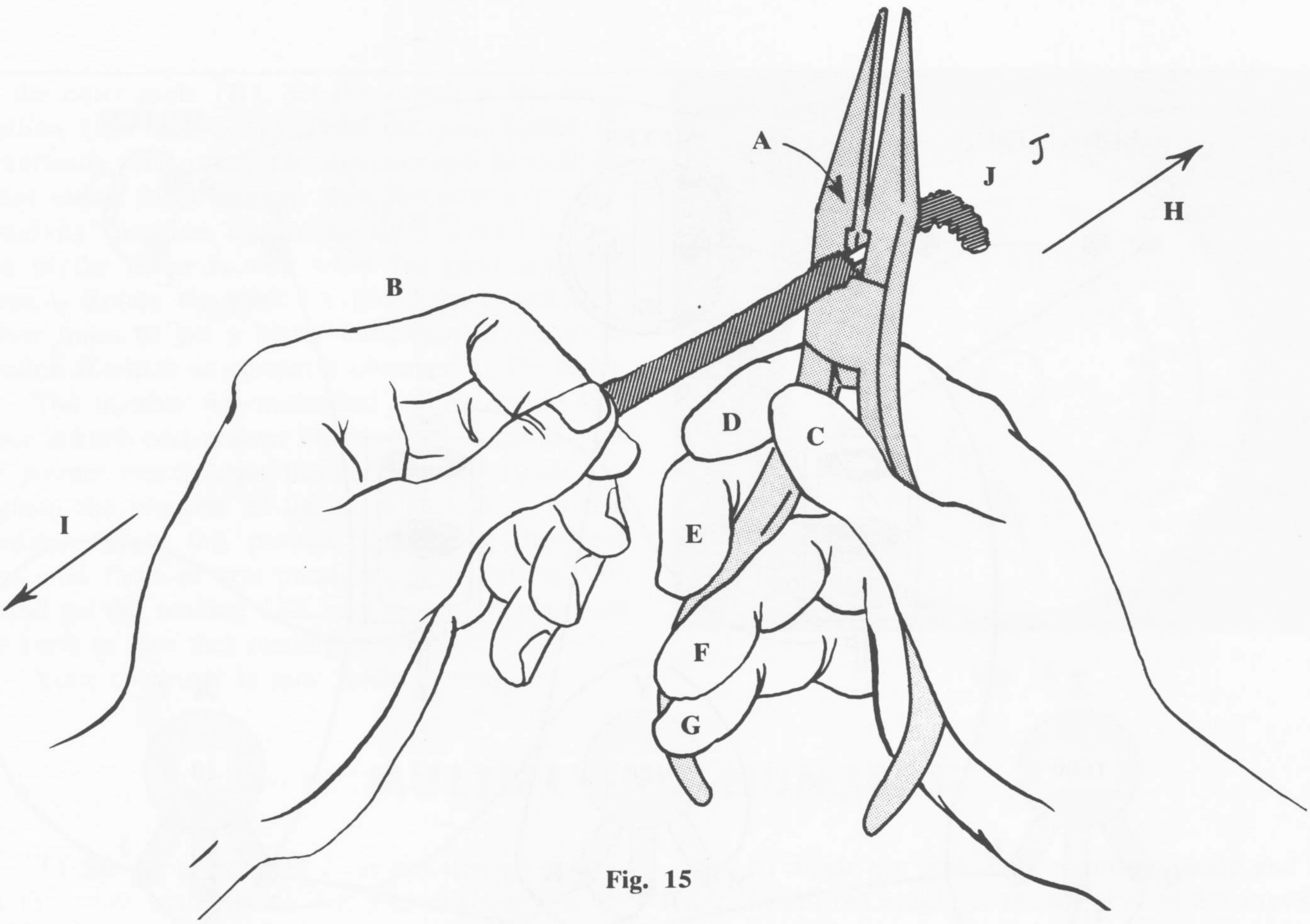


Fig. 15

in Fig. 17. The positions of the wires in the diagram are spread out somewhat to make the connections easier to follow. Make the wires as short as possible, but avoid contact with other metal parts.

While working with the computer box in an inverted position, be sure that it is placed on a clean, dry surface to avoid staining the printed front of the box. It is a good idea to place it on a large sheet of clean cardboard.

13) Your kit contains a small *resistor* (A in Fig. 17). Clean the end of its wire with steel wool to remove any wax or grease. Connect either end to terminal B (of the switch) and the other end to Terminal C (Fig. 17). Connect a wire from B to D. Place nuts on the bolts at B, C, D, and tighten firmly.

14) Connect a wire from the push button at E (either bolt) to the (+) of the flashlight cell at F. Connect a wire from the (—) end of the cell (G) to the (+) end of the second cell (H). Connect a wire from the (—) end of the second cell (I) to terminal J of the 10 ohm pot (X). Tighten with nuts at E, F, G, H, and I, but not at J.

15) Connect terminals J, K, and L of the 3 pots. Tighten with nuts. When tightening the terminals on the pots, avoid too much force, because this tends to twist the strips.

16) Connect terminals M and N. Tighten with nuts.

17) Connect one wire from terminal O of the 10 ohm pot (X) to terminal P of the switch. Tighten terminal P with a nut. Connect a wire from terminal O to Q. Tighten both terminals with nuts.

18) Connect terminal R of the 100 ohm pot (Y) to S of the meter. Tighten both ends firmly.

19) Connect a wire from T of the meter to U of the 1000 ohm pot (Z). Tighten both ends firmly.

20) Check all terminals for firm contact. Place gummed tape across wires to keep them from moving about unnecessarily when the computer is moved. Insert the computer box into the larger outside box (as they came originally in your kit).

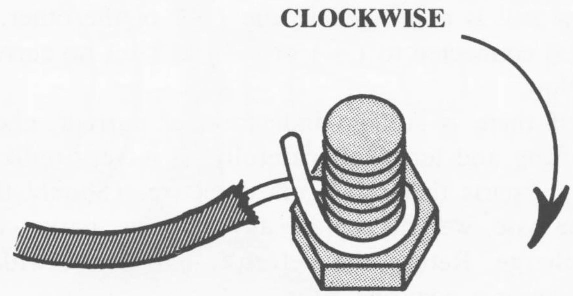


Fig. 16

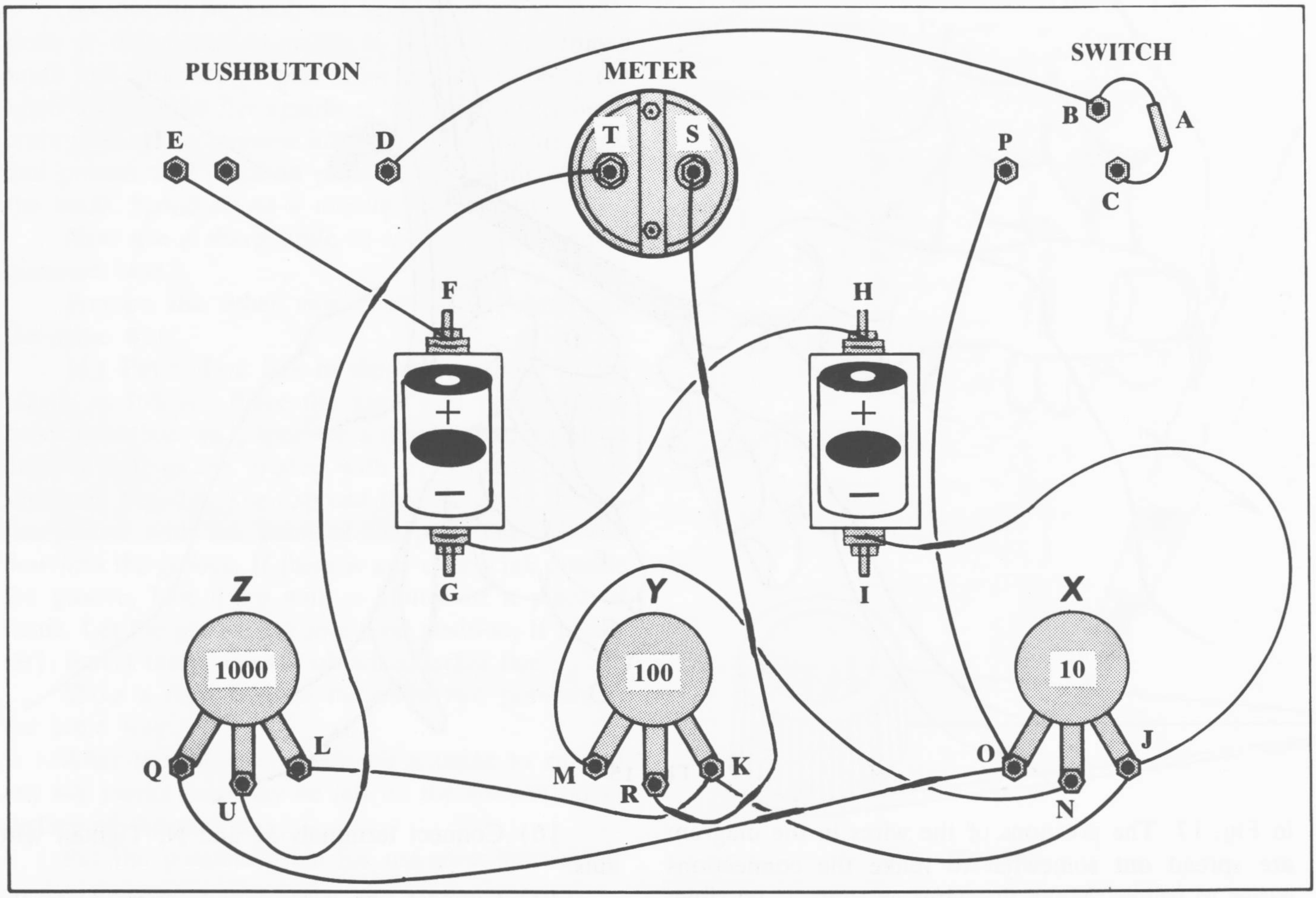


Fig. 17

ALIGNMENT OF POINTERS

Your computer is now ready for final alignment of the pointers. But first check to see that the circuit works properly. Turn the shafts of the 3 pots from the front of the computer. Turn the switch to "low sensitivity" position. Press the push button. Turn any of the pot shafts. If the needle moves, the circuit is probably wired properly. If not, check the flashlight-cell positions in each box to be sure that the (+) of one cell is connected to the (−) of the other. If (+) is connected to (+) or (−) to (−) no current will flow.

If there is still no indication of current, check all wiring and terminals carefully. It is very unlikely that the parts themselves are defective. (Should that be the case, we will replace any defective parts without charge. Return the defective part to us with a request for a new one.)

21) Note the positions of pots X, Y, and Z on the front of the computer box. Turn the *shafts* of all pots to the left (counterclockwise) as far as they will go. Slide a knob and pointer onto pot X and rotate the knob (always counterclockwise) until the center line of the pointer lines up with the line marked "Set Pointer" on the printed computer dial (as shown at A in Fig. 18).

Now tighten the setscrew with a small screwdriver until it grips the shaft tightly. Test the position of the pointer by rotating it back and forth a few times. It should always stop at the "Set Pointer" line when turned to the left. If it does not, adjust the knob and setscrew more carefully.

Do the same thing for pot Y with another knob and pointer.

22) Now set the center line of the pointer for pot X at 6.0 on the outer set of numbers (Scale A on the computer). Set the pointer for pot Y at 70

on the outer scale (B). Set the switch at the lower position (low sensitivity). Press the push button intermittently, and rotate the shaft for pot Z until the meter shows slight motion. Shift the switch to "high sensitivity" position, and rotate shaft Z until no motion of the meter is seen when the push button is pressed. Rotate the shaft for pot Z back and forth a few times to get a better indication of the exact position at which no current is observed on the meter.

The number 6.0 multiplied by 70 equals 420. Place a knob and pointer on the shaft of pot Z with the pointer exactly over the number 420 (scale C). Tighten the setscrew of the knob to lock it in that position. Check this position by turning the knob back and forth as you press the push button. You should get the reading 420, very closely. If not, reset the knob to give that reading.

Your computer is now ready to use.

Fig. 18

MULTIPLYING NUMBERS

1) Set the pointer for X at any desired number on the outer scale (scale A). For example, set the pointer at 5.1 (Fig. 19).

2) Set the pointer for Y at any number on the outer scale (scale B). For example, set the pointer at 42 (Fig. 19).

3) Set the switch at low sensitivity.

4) Press the push button intermittently and rotate the pointer for Z until the needle of the meter moves but slightly.

5) Shift the switch to high sensitivity position.

6) Press the push button intermittently and rotate Z until the needle of the meter does not move. It is a good idea to move the knob at Z back and forth a few times to get a better indication of this position.

7) Read the answer for the multiplication of the readings on X and Y on the outer scale of Z (scale C). In other words, when you are using computer scales A, B, and C, then:

$$X \times Y = Z$$

Your answer for the number above will be close to the correct answer, 214. (See Fig. 19.)

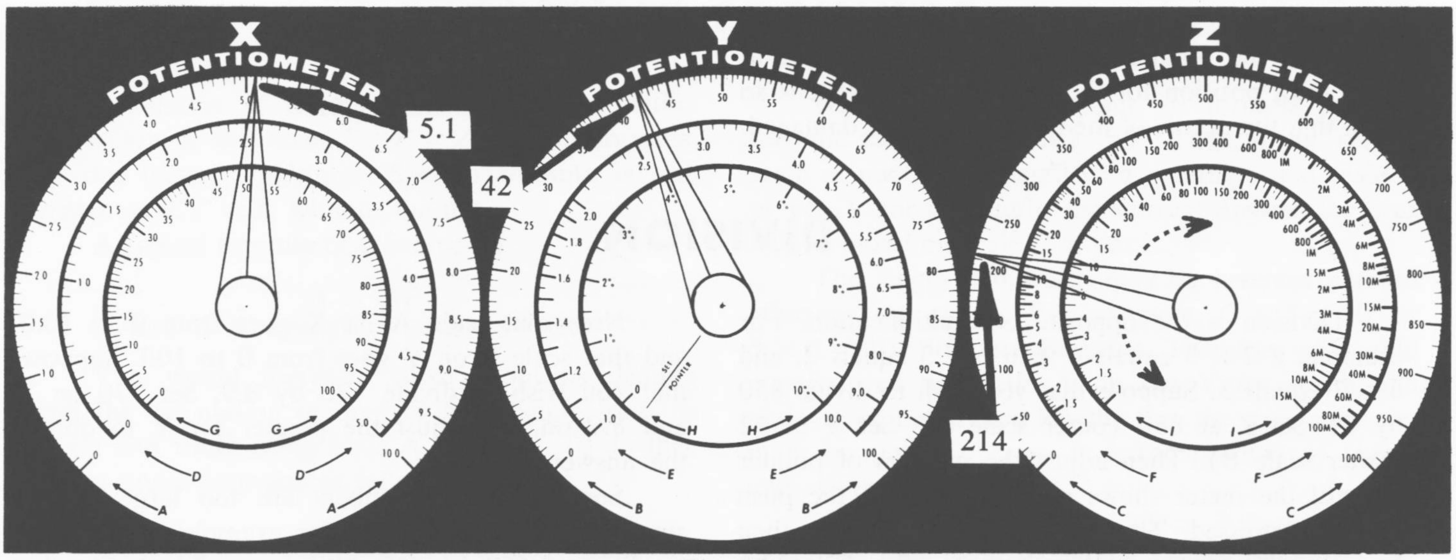


Fig. 19

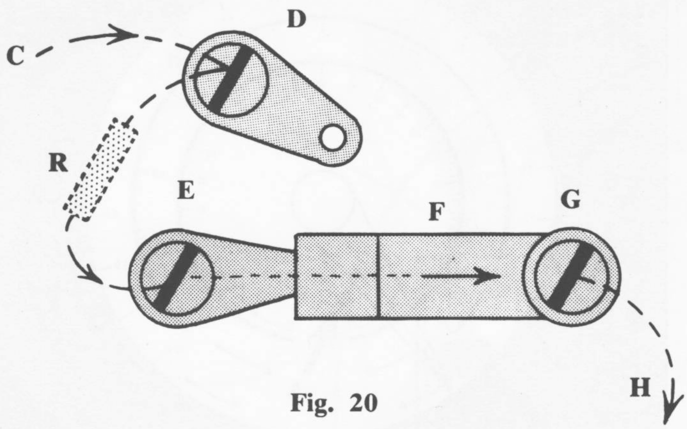


Fig. 20

Always follow these basic steps in finding the position for a correct answer.

The purpose of starting with a low sensitivity position is to prevent possible damage to the meter. Fig. 20 shows the switch from the top of the computer box. The wires and resistor (R) are shown in dotted lines because they are underneath the box. Fig. 21 shows the position of the switch arm when it is in contact with the low sensitivity terminal (E). The current (shown by arrows) then flows from C to D, through R to E, across the switch arm F and out through G and H.

The extra electrical resistance of R reduces the current to a safe amount regardless of the positions of the pointers on each pot. Without this reduction, there are some positions of the pointers at which excessive current may flow in the meter. Although damage is not likely, it is best to avoid any excessive current.

Once the pointers have been adjusted close to the correct position for the answer, the current is so small that the needle of the meter cannot be damaged.

It is then safe to shift to the high sensitivity position (Fig. 21). Then the current flows directly from I to J through K and out through L without reduction by the resistor (R).

After you have become accustomed to handling the computer, you will probably find it possible to operate it at all times on high sensitivity position without having the needle swing beyond the scale of the meter. This may be accomplished by giving the push button rapid but gentle taps so that current flows only momentarily. But at the beginning, until you learn how to do this, it is wise to start with the low sensitivity position.

It also does no harm to keep the push button pressed down continuously once the current is not excessive and the needle stays within the area of the numbers on the dial. However, you will find that it helps in obtaining a more accurate final reading to tap the push button intermittently. Slight motion of the needle then reveals that the dial is not yet at final position for the answer. Only when you detect no motion of the needle at all should you consider the position to be the right one.

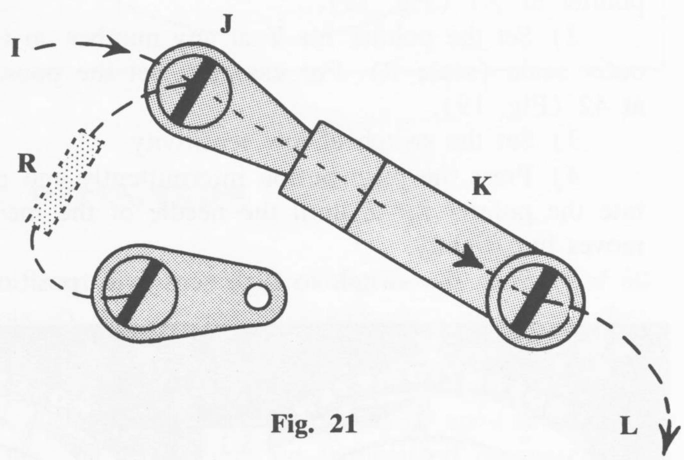


Fig. 21

DIVISION

Division is the opposite of multiplication. For example, if 2 × 3 equals 6 then 6 ÷ 3 equals 2, and 6 ÷ 2 equals 3. Suppose that you wish to divide 850 by 72. Set Z at 850 (outer scale C). Set Y at 72 (outer scale B). Then adjust the position of pointer X until the meter shows no current when the push button is pressed. The answer for 850 ÷ 72 is then found on the outer scale of pot X. (See Fig. 22.)

Note that scale A on X goes from 0 to 10.0 and that scale B on Y goes from 0 to 100. Suppose that you wish to divide 770 by 8.9. Set 770 on Z and 8.9 on X. Adjust the pointer for Y to obtain the answer.

Sometimes the numbers are too large or too small for the scale shown. For example, if you wish to divide 8,650 by 6.5, it is impossible to do this

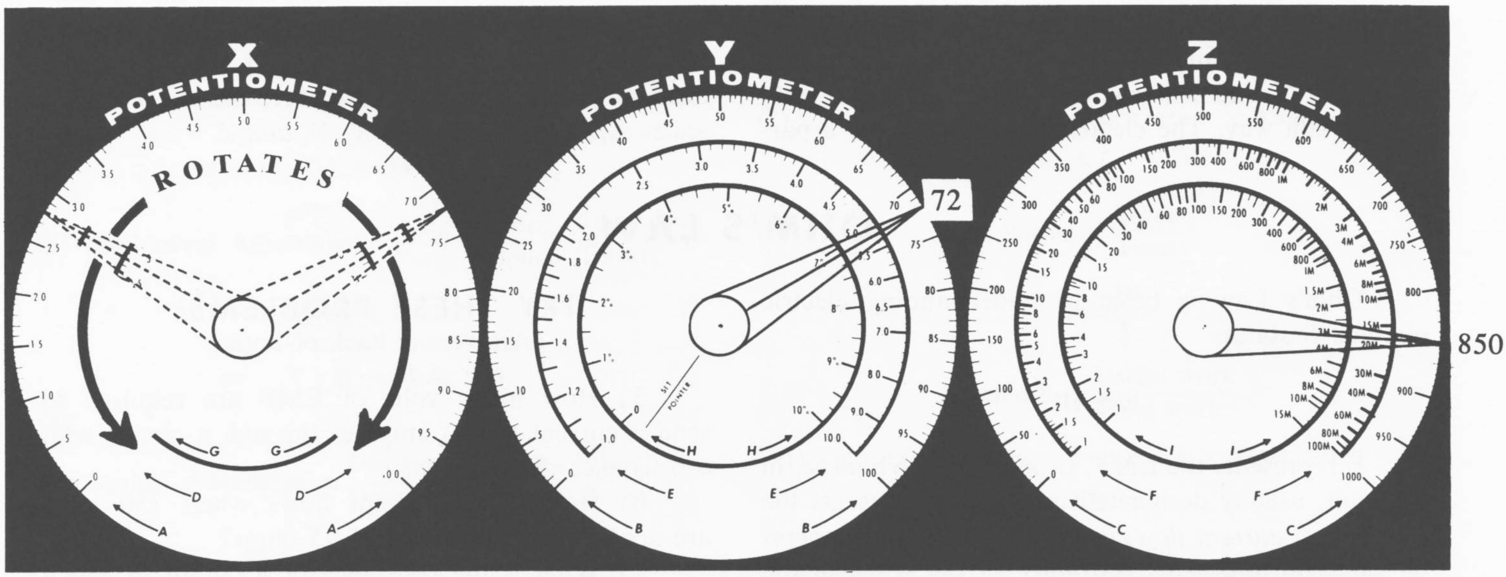


Fig. 22

directly, because the highest number on scale Z is 1,000. In that case reduce the number 8,650 by shifting the decimal point one place to the left (dividing by 10). Now divide 865 by 6.5. When you get the answer, it is 1/10th of the proper amount. Simply shift the decimal point back again to increase the answer 10 times. It will now be the correct answer for the original problem (8,650 ÷ 6.5).

By such shifting of decimal points, it is possible to adjust any multiplication or division problem to fit the numbers on the dials.

Note that the settings of the dial are accurate only to about two or three digits. For a number such as 962.4, position the pointer to the closest setting you can make. Answers will not be more accurate than 2 digits. The question of accuracy is discussed in a later section.

TRY THESE PROBLEMS:

(Answers at back of book)

1) 963 ÷ 41
2) 8.6 × 92
3) 7,520 ÷ 812
4) 62 × 7.9

SOLVING FORMULAS

A number of formulas in science and mathematics are of the form $X \cdot Y = Z$. In this formula, the dot means "multiply." Such a formula is also written as $XY = Z$, without the dot.

A typical formula of this type is:

A = LW
Area of a rectangle = Length × Width

You would therefore solve problems dealing with the calculation of area of a rectangle from its length and width using scale A, B, or C. To do this, first compare formulas as follows:

XY = Z
LW = A

Notice that we have put the formula for area into the same form as $XY = Z$. Thus, if you set the scale at pot X for length of a rectangle and scale B of pot Y for its width, the correct answer for area will be found on scale C of pot Z.

The length and width may be reversed without affecting the result. For example, 2 × 5 = 10 or 5 × 2 = 10. Both are the same. Therefore you could equally well use scale A on pot Y for width and scale B of pot Y for length. But the area must remain on pot Z because it is the product of length and width.

Note that in the formula $XY = Z$ there are three quantities that *vary* or change. These quantities are

therefore called *variables*. But they are related in a certain way in the formula $XY = Z$. This relationship is called a *function*. In this case, Z is a function of X and Y and will change with X or Y but only in a certain way. The electric circuit is set for a particular function in which $Z = XY$. How this is done will be explained later.

Here are a number of formulas that are of the form $XY = Z$ and may therefore be solved with scales A, B, and C of pots *X, Y, and Z.*

OHM'S LAW

Ohm's Law is basic to understanding electric circuits. It states:

$$\mathbf{E = IR}$$

E represents the *EMF,* or *electromotive force,* in a circuit, usually designated in *volts. I* represents the *intensity of current flow,* usually measured in *amperes* or *amps. R* represents *resistance* of the wires in the circuit, usually measured in *ohms.*

TRY THESE PROBLEMS:

(Answers at back of book)

5) How many volts of EMF are required to send a current of 4.2 amperes through a circuit with a resistance of 73 ohms?

6) How much current flows when 750 volts are applied to a resistance of 95 ohms?

7) What is the resistance of a circuit in which 4.5 amps. are caused by 255 volts?

WORK

The basic formula for accomplishment of physical *work* is:

$$\mathbf{W = FD}$$

W represents the amount of work (often expressed in foot-pounds). *F* represents *force* and *D* represents *distance*. In the problems below, we shall express force in pounds and distance in feet. Try the following problems:

8) How many foot-pounds of work are done when a 7.8-pound weight is lifted 82 feet?

9) If 320 foot-pounds of work were done in pushing an object 9.3 feet, how much force was exerted?

PRESSURE UNDER A LIQUID

As one goes under water, the pressure (P) of the water increases with the height (H) of the water above. In liquids other than water, the density (D) of the liquid affects the pressure. The three quantities are related as follows:

$$\mathbf{P = HD}$$

$$\text{Pressure} = \text{Height} \times \text{Density}$$

TRY THESE PROBLEMS:

(Answers at back of book)

10) What is the pressure 9.2 feet under water if the density of water is 62.5 pounds per cubic foot?

11) At what depth will sea water (density of 64 pounds per cubic foot) produce a pressure of 450 pounds per square foot?

12) How dense is a liquid if a pressure of 650 grams per square centimeter is produced at a depth under the surface of 94 centimeters?

ADDITIONAL FORMULAS

Solvable with Scales A, B, C

(1) $\text{Velocity} = \dfrac{\text{Distance}}{\text{Time}}$ $\qquad \mathbf{V = \dfrac{D}{T}}$ *or* $\mathbf{VT = D}$

(2) $$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\mathbf{D} = \frac{\mathbf{M}}{\mathbf{V}}$$

or $$\mathbf{DV} = \mathbf{M}$$

(3) $$\text{Mechanical Advantage} = \frac{\text{Weight Lifted (Resistance)}}{\text{Force Applied (Effort)}}$$

$$\mathbf{M.A.} = \frac{\mathbf{R}}{\mathbf{E}}$$

or $$\mathbf{R} = \mathbf{M.A.} \times \mathbf{E}$$

(4)

$$\text{Mechanical Advantage} = \frac{\text{Distance Moved by Effort}}{\text{Distance Moved by Resistance}}$$

$$\mathbf{M.A.} = \frac{\mathbf{De}}{\mathbf{Dr}}$$

or $$\mathbf{M.A.} \times \mathbf{Dr} = \mathbf{De}$$

(5) $$\text{Specific Gravity} = \frac{\text{Density of Substance}}{\text{Density of Water}}$$

(6) Hooke's Law

$$\text{Elastic Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

(7) Coefficient of Friction on a Level Surface

$$= \frac{\text{Force of Friction}}{\text{Weight}}$$

(8) $$\text{Final Velocity} = \text{Acceleration} \times \text{Time}$$

$$\mathbf{V_f} = \mathbf{AT}$$

(9) $$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$\mathbf{P} = \frac{\mathbf{W}}{\mathbf{T}}$$

(10) $$\text{Efficiency} = \frac{\text{Useful work}}{\text{Total work}}$$

(11) $$\text{Velocity of a Wave} = \text{Frequency} \times \text{Wavelength}$$

$$\mathbf{V} = \mathbf{FW}$$

(12) Electrical Power in Watts $= E \times I$,
(where E is EMF in volts and I is current in amperes)

(13) Area of Parallelogram $=$ Base $\times$ Altitude

(14) Volume of a Prism $=$ Area of Base $\times$ Altitude

Problems dealing with any of the above relationships may be solved with scales A, B, and C of your computer.

HOW DOES THE COMPUTER WORK?

A *potentiometer* is an electrical device that provides a means of obtaining different "potentials" from a given source of electricity. The word "potential" refers to *potential energy* and is measured by the familiar units of *volts*. When we say that one battery has 20 volts and another has 10 volts this means that its ability to do work (potential energy) on a certain amount of electric charge is twice as great for the 20 volt battery as for the 10 volt battery.

Voltage (or potential) is also referred to as EMF (electromotive force). If a battery has high push (force), it causes more current.

Consider Fig. 23. A is the symbol for a battery with a certain voltage. The (−) terminal has an excess of electrical particles (electrons) which will flow around through a circuit (BCDEFGH) and back to the (+) end of the battery at H, provided there is some kind of conducting (wire) path. Such an electrical path is known as a *complete circuit*.

Some wires have more *resistance* to flow of current than others. Connecting wires C and G are usually of relatively thick copper or aluminum, to offer little resistance to flow of the electrons. The zigzag line E in Fig. 23 is a symbol for resistance. It is wire made of special metal, thin and long, selected in this way to increase resistance. Thus, in the circuit BCDEFGH, almost all the work done by the battery occurs in forcing the electrons to move through the high resistance E, and very little work is done in pushing electrons through the connecting wires C and G or the battery A.

Now suppose we take a "voltmeter," which measures *volts*. We put its two terminals across the battery from the (+) to the (−), from H to B. Assume that it reads 10 volts. Now touch its terminals from D to F. Again it reads 10 volts. Connect the voltmeter from D to B. It reads 0. Connect it from H to F. It reads 0. This means that practically all the voltage of the battery is used in pushing electrons through the high resistance E and practically none in the low resistance conducting wires C and G.

Let us place the terminals of our voltmeter from

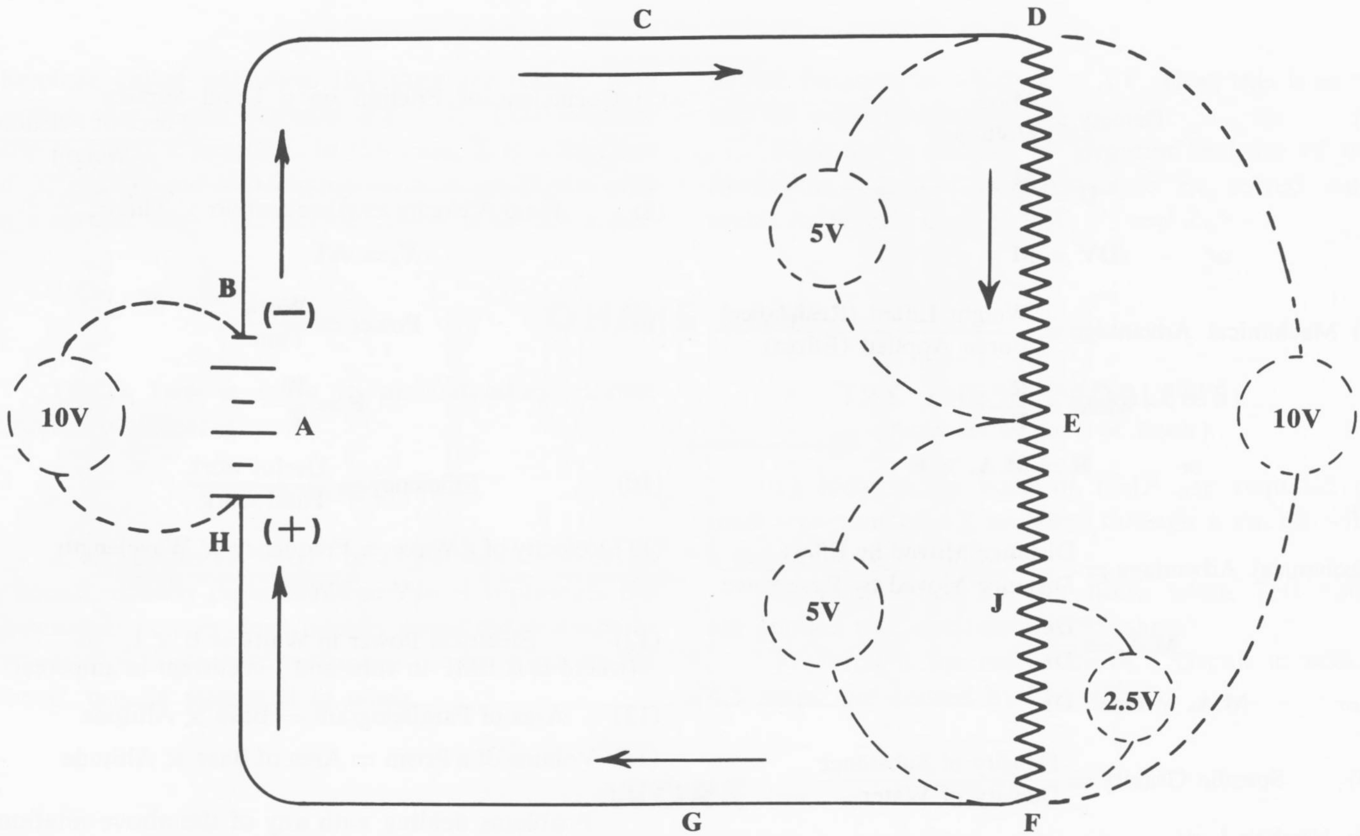


Fig. 23

one end of the high resistance wire (D) to the midway point (E). Now the meter reads 5 volts. Try it from the midway point E to point F. Again the reading is 5 volts. We say that the "voltage drop" or "potential drop" from D to E is 5 volts. The potential drop from E to F is also 5 volts. In other words, half of the voltage is used in pushing electrons through half of the resistance wire and the other half of the voltage is used up in pushing the electrons through the remaining half of the wire.

Suppose we place the voltmeter terminals across a fourth of the length of the wire, from J to F in Fig. 23. Now the reading is 2.5 volts, which is a fourth of 10 volts.

We see that the "voltage drop" is in proportion to the length of the resistance wire across which the voltmeter terminals are placed.

The electric circuit of your analog computer is shown in Fig. 24. The dotted oval marked X represents potentiometer X. The three terminals on the pot are indicated by A, B, and C. A and B, the two outer terminals, are the ends of a long coil of resistance wire. The coil and the end terminals are shown in Fig. 25. A sliding arm (C in Figs. 24 and 25) moves along the entire length of the resistance when the shaft is rotated. As it moves from B toward A, it includes a larger and larger proportion of the wire until at position A it includes the entire wire, or all the resistance.

Potentimeter Y is shown in Fig. 24 with its three terminals D, E, and F. It is shown smaller than pot X merely for clarity in the drawing and not because it really is that way. Potentimeter Z is shown with its three terminals G, H, and J.

Suppose that the position of the rotating part of pot X (C in Fig. 24) is at the halfway point and that the position of the rotating part (F) of pot Y is also at its halfway mark. Terminal C of pot X then takes off half of the voltage drop from A to B. Then terminal F of pot Y takes off half of that. The net result is that the final voltage drop from F to E is $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$ of the original amount.

Suppose we set the rotation point of pot Z (J in Fig. 24) at a fourth of the way from H to G. The voltage drop from J to H will be a fourth that from G to H.

Now note that A and G are connected. So are B, E, and H. This causes the total voltage drop from A to B to be the same as from G to H. Because position J and position F of the rotating arms of pots Y and Z both produce a fourth of the same voltage, the voltage drops from F to E and from J to H are

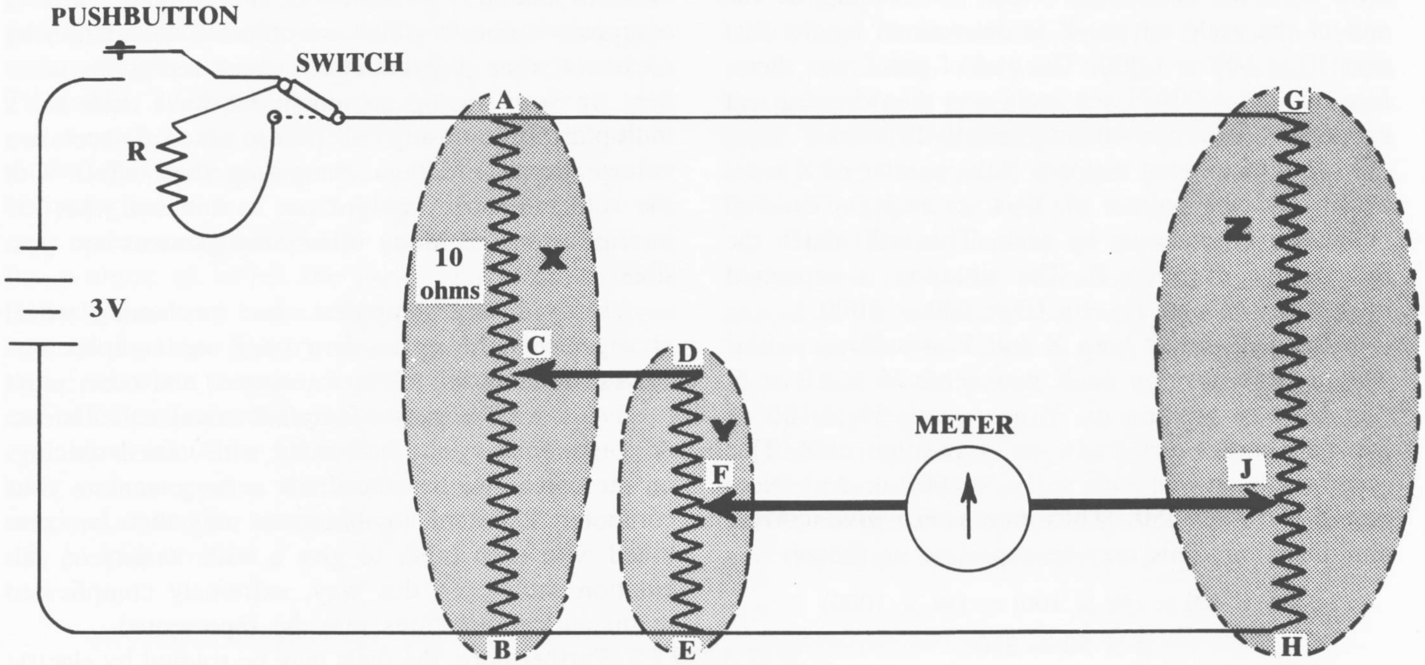


Fig. 24

the same. As a result, there is no flow of current through the meter at that position and the pointer of the meter registers no current by remaining still.

Suppose that you move the pointer at J closer to G. The voltage drop would now be greater than from F to E. This would cause current to flow in the meter and would be indicated by motion of the needle. Similarly, if point J were moved closer to H this would cause the voltage drop from F to E to be higher and current would flow in the meter in the opposite direction. The needle would again move. Thus, current does not flow in the meter when the voltage drop from F to E is the same as from J to H.

Now the voltage drop from F to E was obtained by two successive drops. First the rotating arm C took off one half of the voltage, then F took off half of that. Thus: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The final position of J gives the same answer: $\frac{1}{4}$.

Suppose we set the position of point C to take off $\frac{2}{3}$ of the voltage drop of pot X, while F is set to take off $\frac{3}{4}$. Then the voltage taken off by F would be $\frac{2}{3} \times \frac{3}{4}$ or $\frac{1}{2}$. The setting of J on pot Z to give no current in the meter would then be $\frac{1}{2}$. Thus the position of the rotating arm on pot Z indicates a multiplication when the meter reading is zero.

When we designed the outer scales on each pot, the numbers were marked off around part of a circle from the start (0) to the end of the scale at evenly spaced intervals. Such a scale is called *linear*. This corresponds to the evenly spaced arrangement of the wire in each pot. The end of the scale can be set at any desired number for pots *X* and *Y*. We happened to choose the number 10 for the first scale and 100 for the second. We might just as well have selected 5 and 14.2 or 6.9 and 8,651. However 10 and 100

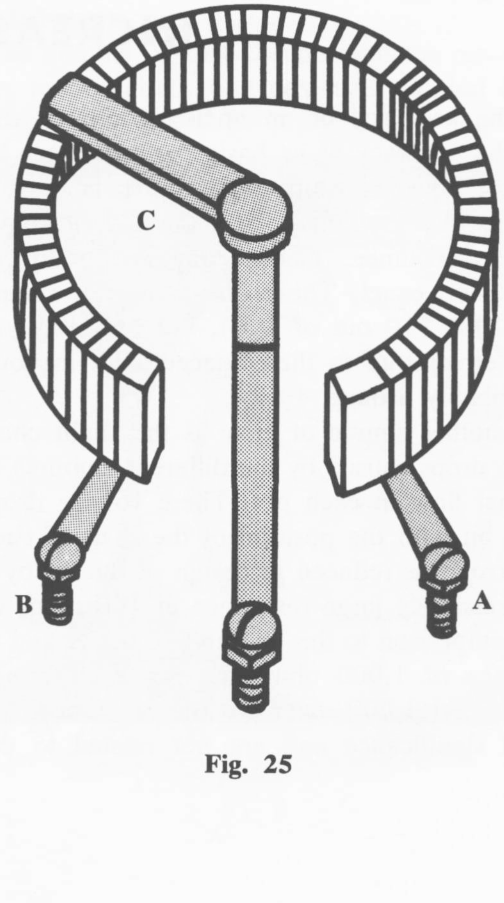


Fig. 25

have obvious advantages. Now the reading of the end of the scale on pot Z is determined by the fact that $10 \times 100 = 1{,}000$. The end of pot Z was therefore marked 1,000. Each scale was then divided and subdivided into conveniently small divisions.

Now note what happens if the pointer of X is set at 10, and the pointer of Y is set at 100. The full voltage drop is shown by each. This will match the full voltage drop on Z. The situation is expressed by $1 \times 1 = 1$ and also by $10 \times 100 = 1000$.

Suppose we set both X and Y at halfway points. The halfway position on X now reads $\frac{1}{2} \times 10$ or 5. The halfway position on Y now reads $\frac{1}{2} \times 100$ or 50. This causes a $\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$ position of Z. This corresponds on our scale to $\frac{1}{4} \times 1000$ or 250. Note that 5×50 is 250. Thus our scales give correct multiplication. This may be expressed as follows:

$$(\tfrac{1}{2} \times 10) \times (\tfrac{1}{2} \times 100) = (\tfrac{1}{4} \times 1000)$$

$$5 \times 50 = 250$$

Finally, what happens if we move the pointers of X and Y to zero? In that case, there is no voltage drop from F to E (Fig. 24). Pointer J on pot Z must be moved to the zero position to produce the same voltage drop, as indicated by the meter. We now have $0 \times 0 = 0$. Thus, all positions of Z give multiplications of X and Y when the meter reads zero.

An *analog* is a likeness or similarity. An analog computer is one in which we obtain a calculation by means of some physical effect which represents numbers. In your analog computer, we have achieved a multiplication by using two pots to take off successive voltage drops and then comparing their effect with the voltage drop in a third pot to find out what the multiplication is. Many other arrangements are possible.

Early analog computers were mechanical wheel arrangements. Many modern ones use complex circuits of resistances, tubes, transistors, and other parts to give a wide variety of mathematical calculations. A potentiometer may be wound with varied spacings on the coil instead of the linear arrangement in your computer. Different combinations may then be combined with each other to give a wide variety of calculation setups. In this way, extremely complicated mathematical functions may be represented.

Furthermore, the dials may be rotated by electric motors and gear combinations rather than by hand. The computer can then be made to indicate calculations automatically. Or the entire arrangement may be completely electronic with almost no moving parts, in which case the rapidity and complexity of the calculation is limited mainly by the ability and creativeness of the designers and operators of the computers.

INCREASING ACCURACY

The accuracy of an analog computer depends upon the accuracy of its basic parts. In your kit, the distances between windings of the pots may vary a bit. The wire itself may vary, thereby causing variations in resistance. The starting and ending points are not too exact. These errors may in some cases reach 10% (10 out of 100), but only rarely. More likely, errors due to these inaccuracies in your pots are probably around 4%.

Another source of error is the slight change in voltage drops caused by the different amounts of current that flow in each pot. These voltage drops will vary a bit with the position of the shaft of each pot. This error was reduced in design of the kit by selecting a relatively large resistance of 100 ohms for pot Y in comparison to the 10 ohms of pot X and a large resistance of 1,000 ohms for pot Z. The numbers 10,100, and 1,000 ohms for the resistances have no special significance and are not related to the fact that we chose 10, 100, and 1,000 as the values for the scales. They simply happened to be convenient amounts of resistance to use.

The error due to the effect of flow of current in different parts of the circuit is at most 2½%. All told, it is probable that your computer produces errors on scales A, B, and C of 6% or less. Thus, if an exact answer should be 500, it is possible that it may vary by 6% of 500, or 30 more or less (from 470 to 530).

Some of the error was reduced by making your original setting of the dial at pot Z for a particular problem. You will recall that you set dial X at 6.0 and Y at 70 and then set the dial Z to 420 when the meter read zero. These numbers were selected because they are approximately at the center of the scales. An extreme error at one end of the scale or the other is therefore halved.

You might now try to reduce any error that

arises from an incorrect zero setting. The "Set Pointer" line is not necessarily exact. Try the following procedure to get a better zero setting on pots X and Y. Turn dial Z counterclockwise all the way to zero and beyond to the point where it stops. Turn pot Y to 100 (extreme right). Now the only position of X that gives a zero reading on the meter is at the zero resistance. Turn pot X down to zero and note the position at which the meter shows no current. Actually you may find a slight jiggle all the way from zero to the stop. But try to estimate when the slight jiggle starts to increase. Loosen the setscrew of the dial at X and reset it to the new zero.

Now set X to 10 and Z to zero. Turn Y to zero and note when the meter current starts to increase. Loosen the setscrew and set the Y pointer at this position to zero.

Finally, set X and Y at definite numbers, such as 6.0 and 70. Rotate the shaft for Z until the meter reads zero when the push button is pressed. Reset the pointer for Z to read 420 at that position. Experiment with several multiplication problems. Has accuracy been increased? By this procedure you may be able to approach 2 or 3% accuracy.

Still further increase in accuracy entails much higher cost for parts. Potentiometers that are guaranteed at 2% accuracy cost more than three times as much as the ones used in your kit. More sensitive meters that would permit more accurate settings also go up in price very rapidly. A point is soon reached at which it no longer pays to increase accuracy any further. As indicated earlier, this limit today is about .01%, but new developments are constantly occurring, and this figure will probably be surpassed.

POWERS

If you multiply the number 3 by itself, you obtain: 3×3 or 9. This can also be written as $3^2 = 9$. The small 2 to the upper right of 3 is called a *power*. It informs you that the number to the left of it is to be multiplied by itself twice (counting the original number as once).

What does 3^4 mean? Mathematicians have agreed to let this represent $3 \times 3 \times 3 \times 3$. You can see by multiplying that the answer is 81. Thus: $3^4 = 81$. This is read as "3 to the 4th power equals 81."

The second set of numbers (scales D, E, and F) on dials X, Y, and Z enable you to estimate powers of a number. For example, find 5^4 (read: 5 to the 4th power). This is equal to $5 \times 5 \times 5 \times 5$.

Set the power on scale D of pot X to number 4.0. Set the number 5 on scale E of pot Y (Fig. 26). Rotate the Z dial and find the answer on scale F of pot Z when the meter does not move as you press the push button.

The number on dial D represents the power. The number to be raised to a power is on dial E. The answer is found on dial F.

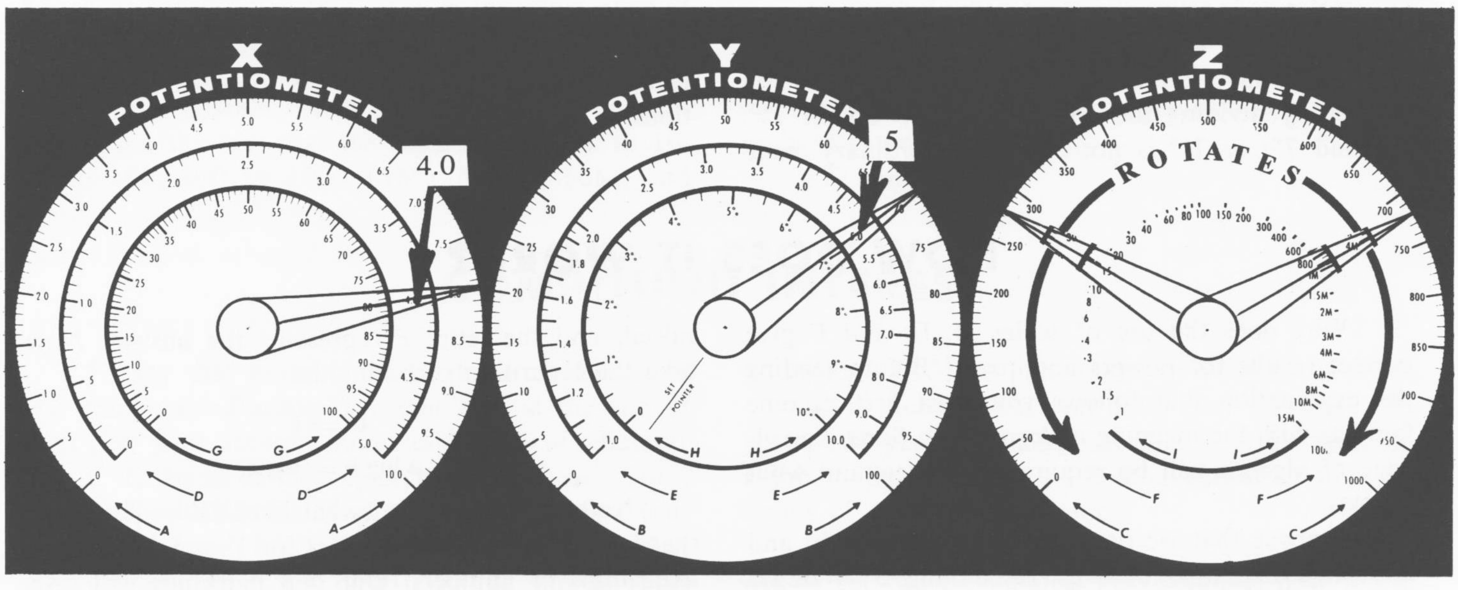


Fig. 26

The correct answer to 5^4 (or $5 \times 5 \times 5 \times 5$) is 625. Your computer pointer will be near this number, but it will be difficult to estimate accurately. Notice that the numbers on dial F increase very rapidly, and even a slight shift in position causes a big difference in the answer. Because of the large errors on this scale your answers are rough estimates.

Estimates of this kind can be very useful in solving such a problem as this one: Find $6.3^{3.5}$. Set the power (3.5) on scale D. Set 6.3 on scale E. Find the answer on scale F. The correct answer is 396. You will find the pointer on scale F somewhere in that region.

But you must be wondering what $6.3^{3.5}$ really means. The next section will help you understand it.

ROOTS

In the previous section you learned that $5 \times 5 \times 5 \times 5$ (5^4) equals 625. Suppose you were asked to find a number whose 4th power is 625.

You know that 5 is the answer, since $5 \times 5 \times 5 \times 5$ equals 625. The number 5 is said to be the fourth *root* of 625. To find this number with your computer set the number 4 (the root) on dial D. Set 625 on dial F and then rotate the center dial and read the answer (5) on dial E. You simply reverse the procedure to find the root.

Mathematicians write a root in this manner $\sqrt[4]{625}$. It may also be written as $625^{1/4}$. The second root of a number is usually called the *square root*. The third root of a number is usually called a *cube root*.

Try this problem. Find the 4th root of 7 raised to the 18th power. This means that you are to find a number which, when multiplied by itself 4 times, equals 7 multiplied by itself 18 times. Or this could be written as:

$$\sqrt[4]{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}.$$

(Note: The dots represent multiplication.)

We could write this more simply as $7^{18/4}$. Mathematicians have found it possible to simplify this still further by dividing the 18 by the 4, to obtain $7^{4.5}$. To find $7^{18/4}$ or $7^{4.5}$ is not easy using ordinary arithmetic. With your analog computer, a quick estimate can be obtained. Set the power (4.5) on scale D. Set the number 7 on scale E. Find the estimated answer on scale F. The correct answer is 6,353. You will find your answer on the computer somewhere near it. (Note: to reduce the use of zeros on scale F the letter M is used to denote thousand. Thus 30M means 30,000.)

Suppose the problem were to find $\sqrt[4.5]{6{,}353}$ (find the 4.5 root of 6,353). You simply work backward. Set 4.5 on scale D. Set 6,353 (approximately) on scale F. Find the answer on scale E (approximately).

Now try this problem. Find $(8.2)^{4.4}$. Set 4.4 on scale D. Set 8.2 on scale E. Find the answer in the usual way on scale F.

What does $(8.2)^{4.4}$ mean? It could mean $(8.2)^{44/10}$ or $(8.2)^{22/5}$ or $(8.2)^{88/20}$. All of these amount to the same thing and have the same answer. But, for example, $(8.2)^{22/5}$ means: multiply 8.2 by itself 22 times and then take the 5th root of the result. Isn't it amazing that you can get an estimate so quickly with your computer? This gives you some idea of the ability of computers to speed up calculations.

HOW DOES IT WORK?

Why does the use of scales D, E, and F give correct results for powers and roots? Before reading the explanation that follows you must first become familiar with the meaning of *logarithms*. Some knowledge of algebra will be required to understand what follows.

Suppose that we have three numbers, a, b, and c, related like this: $b^a = c$ (resembling $5^4 = 625$). The letter a represents the power to which b is to be raised, and the letter c represents the answer. Now take the logarithm of both sides.

$$\mathbf{b^a = c}$$

$$\mathbf{a \log b = \log c}$$

Note that this is somewhat like $XY = Z$, except that instead of regular numbers for Y and Z we have logarithms of numbers. Our dial markings will give correct results if scales E and F (on the dials for

pots Y and Z) are marked off in logarithms rather than in regular numbers. When we designed scales D, E, and F, it was decided to handle powers only to 5. Note that scale D is marked off equally around the circle from 0 to 5. Observe that the numbers on scale D are exactly half those on scale A.

It was decided to take powers only of numbers from 1 to 10. Therefore the highest point on scale E was set at 10.0. But observe that the lowest point on scale E is 1.0, not 0 as you might expect. The same thing is true of scale F. The reason for this is that:

$$1^0 = 1$$

$$0 \log 1 = \log 1$$

and, since log 1 is 0,

$$0 \times 0 = 0$$

(which represents the actual relationship between X, Y and Z when all are at 0).

Thus, if we set the positions of pots X, Y, and Z all at zero, the correct power (on scale D) is 0, the number being raised to a power (on scale E) is 1, and the answer (on scale F) is 1.

Now note that the scale on E is a peculiar one in that the spacings between numbers shrink as the numbers increase from 1 to 10. This scale is "logarithmic." In fact, it is identical to the scale on a slide rule. Thus, when you set a number on scale E, you are really setting its logarithm and not the number itself.

Place the pointer of pot Y at 1.0 on the E scale. Rotate it to 2.0. Then rotate it to 4.0, then to 8.0. You will observe that there is the same amount (angle) of rotation from 1 to 2 as from 2 to 4, as from 4 to 8. This is the nature of a logarithmic scale.

Similarly, scale F is logarithmic, as you can observe from the fact that the rotations from 1 to 10, 10 to 100, 100 to 1000 (1M), 1M to 10M, and 10M to 100M are all the same. These numbers are 10^1, 10^2, 10^3, 10^4, 10^5.

Turn the pointer of pot Z to 10 on scale F. Note that 10 (or 10^1) is in the same position of the pointer as the number 200 on scale C. The number 100 (or 10^2) is in the position for 400. The number 1000 (or 10^3) is in the position for 600. The number 10,000 (or 10^4) is in the position for 800. The number 100,000 (or 10^5) is in the position for 1000. Thus the positions of the numbers of scale F are logarithmic, as is the case of scale E of pot Y. Thus we get answers for $a \times \log b = \log c$, which also gives us answers for $b^a = c$.

In designing the markings for scale F, the number 100,000 was placed at the end of the scale because if pot X is set at its highest value for scale D (at the number 5.0) and pot Y is set at its highest value (at the number 10.0), this represents 10^5 or 100,000. The correct position of the dial on pot Z should then be at the end of the scale. Therefore the end of the scale is labeled 100M (or 100,000). Scale F was then subdivided into 5 equal parts (for powers 1, 2, 3, 4, 5), representing 10^1, 10^2, 10^3, 10^4, and 10^5. The scales for 1 to 10, 10 to 100, etc., were then marked off logarithmically, in a manner similar to the scale E on pot Y, but shrunk to 1/5 size.

TRY THESE PROBLEMS:

(Answers at back of book)

13) Find $6.3^{3.9}$.

14) What is the cube (third) root of 4.9 to the 14th power?

15) Find the 4th root of 3,200.

16) What is the square root of 9.3 multiplied by itself 9 times?

17) Find $8.8^{4.1}$.

COMPOUND INTEREST

To say that money in a bank earns interest at 3% compounded annually means that at the end of each year your account increases by 3% of whatever was in the bank at the beginning of the year. Thus, if you deposited \$100.00, at the end of the first year you would have \$103.00 (\$100 + \$3). The next year you would get 3% of \$103.00 as interest. This amounts to \$3.09. Thus at the end of two years you would have \$103.00 plus \$3.09, or \$106.09.

How much would you have after 10 years, 20 years, 100 years? You can see that it would be quite a job to figure it out for a long period of time by means of ordinary arithmetic.

Scales G, H, and I on pots X, Y, and Z enable you to get rapid estimates. For example, how much would \$1.00 amount to in 25 years at 4% interest?

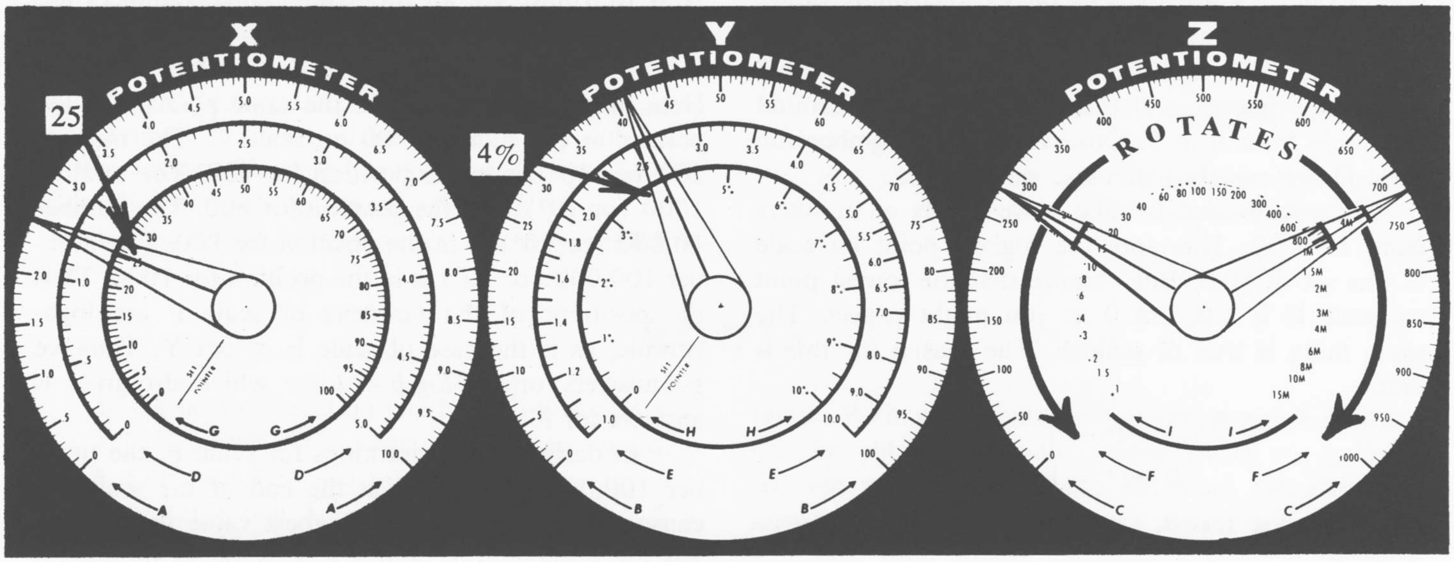


Fig. 27

Set scale G at the number 25 (number of periods, in this case years). Set scale H at 4% interest. The answer is found on scale I in the usual way by rotating the pointer until the meter reads zero when the push button is pressed. In this case, the correct answer is \$2.67. You will find your answer somewhere in that region. (See Fig. 27.)

Note that the calculation is for a starting amount (called the principal) of \$1.00. If you had a starting amount of \$100.00 the final amount would be 100 times \$2.67 or \$267.00.

Now try this problem. What is the amount of \$1.00 in the bank at 4% interest compounded semi-annually for 30 years? In this case, we must find the interest every *half year*. For a half year the interest would be half that for a full year. Then the interest rate would be 2% instead of 4%. But in 30 years we would have to calculate interest 60 times instead of 30 (60 half-year periods).

To solve this problem set dial G on pot X at 60. Set the 2% interest on scale H of pot Y. Find the answer on dial I of pot Z. It is somewhere near \$3.28.

To solve a compound interest problem, set the number of interest periods on scale G of pot X. Set the interest rate of the period on scale H of pot Y. Find the final amount on scale I of pot Z, in the usual manner.

Why does it work?

The formula for the amount (A) of \$1 at an interest rate (i) for a number of periods of time (n) is given by the formula:

$$\mathbf{A = (1 + i)^n}$$

Taking the logarithm of both sides we get:

$$\mathbf{\log A = n \cdot \log (1 + i)}$$

This resembles the function $XY = Z$ in that n multiplied by a function of i, produces a function of A. Therefore we can put it onto our scales provided that we mark them off properly. Scale G of pot X was marked off linearly to represent 100 periods (n). Scale H was marked off according to the logarithms of (1 + i), with interest rates starting at 0 and going up to 10%. Scale I was marked off according to the logarithm of the resulting amount (A). A calculation was made for the value of A that would result from \$1 at 10% for 100 years, and this figure was used as the maximum value for A.

Here is a typical problem:

How long would it take to increase your money 5 times at a rate of interest of 6%, compounded semi-annually? In this case the amount of interest per period (half year) is 3%. Set scale H on pot Y at 3% interest. Set scale I on pot Z at \$5.00 (five times your \$1.00). Rotate pot X and find the answer for the number periods at about 54. This represents 54 half years or 27 full years. Thus your money would increase 5 times in 27 years at 6% compounded semi-annually.

TRY THESE PROBLEMS:

(Answers at back of book)

18) How long would it take to triple your money at 5% compounded annually?

19) Which is better, 6% interest compounded quarterly, or 6½% compounded annually? How much difference would there be on a $1,000 investment for 25 years?

20) What interest rate is required to multiply your money 10 times in 40 years?

21) What will $1.00 amount to at 7% interest compounded semi-annually for 40 years?

GROWTH

Many quantities in nature change in a manner similar to the way interest accumulates in a bank. Among such quantities are: growth of a nation's production, growth of population, development of chain reactions in atomic energy, chemical explosions. All of these situations involve growth or increase of some factor.

In other cases, quantities *decay* or decrease at a certain rate. This is typified by radioactivity and is applied in a practical way in measuring the age of rocks or fossils.

Scales G, H, and I, which you have used for calculating compound interest, may also be used to compute growth. Consider this example. A nation increases in population at the rate of 4% a year. How long would it take for the population to quadruple?

Note that this problem is similar to the problem: How long would it take for money invested at 4% interest to multiply 4 times?

Set pot Y at 4% on scale H. Set pot Z at 4 on scale I. Rotate pot X and find the answer (on scale G of pot X) for the number of years it would take (somewhere around 35 years).

How would you do the following problem? If population is increasing at a rate of 3% per year, how many times as many people will there be after 100 years?

Set scale G of pot X at 100. Set scale H of pot Y at 3%. Find the answer on pot Z (somewhere around 19.2). In other words if the population increases at 3% per year without change for 100 years there would be more than 19 times as many people alive at that time! You can see that such uncontrolled rates of growth of population could be disastrous to mankind.

Here is another type of problem. The increase in production of a nation occurs at the rate of 3.5% per year. How much greater will its production be in 50 years?

Set scale G of pot X at 50. Set scale H on pot Y at 3.5%. Find the answer on scale I of pot Z. The answer appears at about 5.6.

Another problem: Nation A, with an annual increase in production of 6%, produces half as much as nation B, which has an annual increase in production of 4%. How many years will it take for nation A to surpass nation B in production?

Solve for the amount of increase for each nation for several periods such as 20 years, 30 years, etc. Make a table or graph and locate the approximate time when the production of both will be equal.

For example, assume nation A to start with 100 units of production and nation B with 200. Assume 20 years. Use your computer to find the growth for 20 years at 6% (nation A). This turns out to amount to 3.2. Since our dials are based upon a starting point of 1, the production of nation A would be 3.2 times 100 or 320 in 20 years.

Nation B increases at 4%. Solve for 4% at 20 years. The final amount is 2.2. Nation B will then be producing 200 × 2.2 or 440. It is still ahead, but not by much.

Solve in the same way for 30 years. This time we get:

Nation A 570
Nation B 650

Try 35 years:

Nation A 770
Nation B 790

Try 36 years:

Nation A 810
Nation B 820

Try 37 years:

Nation A 860
Nation B 850

Thus nation A will surpass nation B in about 37 years if the rates of growth are maintained.

TRY THESE PROBLEMS:

(Answers at back of book)

22) If a business grows at a rate of 5% a year, how many times bigger will it be in 50 years?

23) General Merchandising Corp. has three times as much in assets as the Premier Mercantile Co.; but PMC is increasing at a rate of 10% a year, whereas GMC is increasing at a rate of 5%. How long will it take for PMC to surpass GMC in assets?

24) The directors of a corporation have the choice of using all of their 8% annual profit for growth, or distributing half of it (4%) as dividends. Compare the growth of the corporation under each plan for 40 years.

RANGE OF A PROJECTILE

There is an extra set of marked dials in your kit. Remove the 3 knobs by loosening the setscrews on each. Place the panel of 3 dials over the ones printed on the computer box (Fig. 28). There are 4 small pointed screws in your kit. Use them to fasten the panel in place, as shown in Fig. 28.

Turn all three pots to the extreme left until they stop. Replace the knobs on each shaft. Set each pointer at the line marked "Set Pointer." They are now in the correct position for operation.

Scales J, K, and L provide computations for the range (R) of a projectile. Fig. 29 shows the path of a projectile shot from a gun at A. The muzzle *velocity* (leaving the gun) of the projectile is V. The angle at which the gun is pointed upward from the ground is θ (the Greek letter "theta"). These two quantities determine the *range* (horizontal distance traveled by the projectile on level ground).

Try this problem: a gun is set at an angle of 30° to the ground. Its projectile has a velocity of 1,500 feet per second. How far forward will it travel before hitting the ground? In other words, what is its range?

Set the angle at 30° on scale J of pot X (Fig. 28). Set the velocity of 1,500 feet per second on scale K of pot Y (Fig. 28). Find the answer in the usual way on scale L of pot Z. The answer is about 61,000 feet.

Try this problem: The projectile of a cannon has a velocity of 1,800 feet per second. At what angle to the ground should it be inclined to strike an object 40,000 feet away?

Set 40,000 on scale L of pot Z. Set 1,800 on scale K of pot Y. Find the answer in the usual way on scale J of pot X. It is about 23°. Note that another answer is possible, 67°. This means that if the gun were pointed high (67°), it would strike the same spot as when pointed low (23°). Notice that the two angles for each point on scale J always add up to 90°. The angles are said to be *complements* of each other. They are related as shown in Fig. 30.

Why does this set of scales give correct answers? (Some knowledge of trigonometry will be required to understand the explanations for all of the functions of the extra set of scales which you have

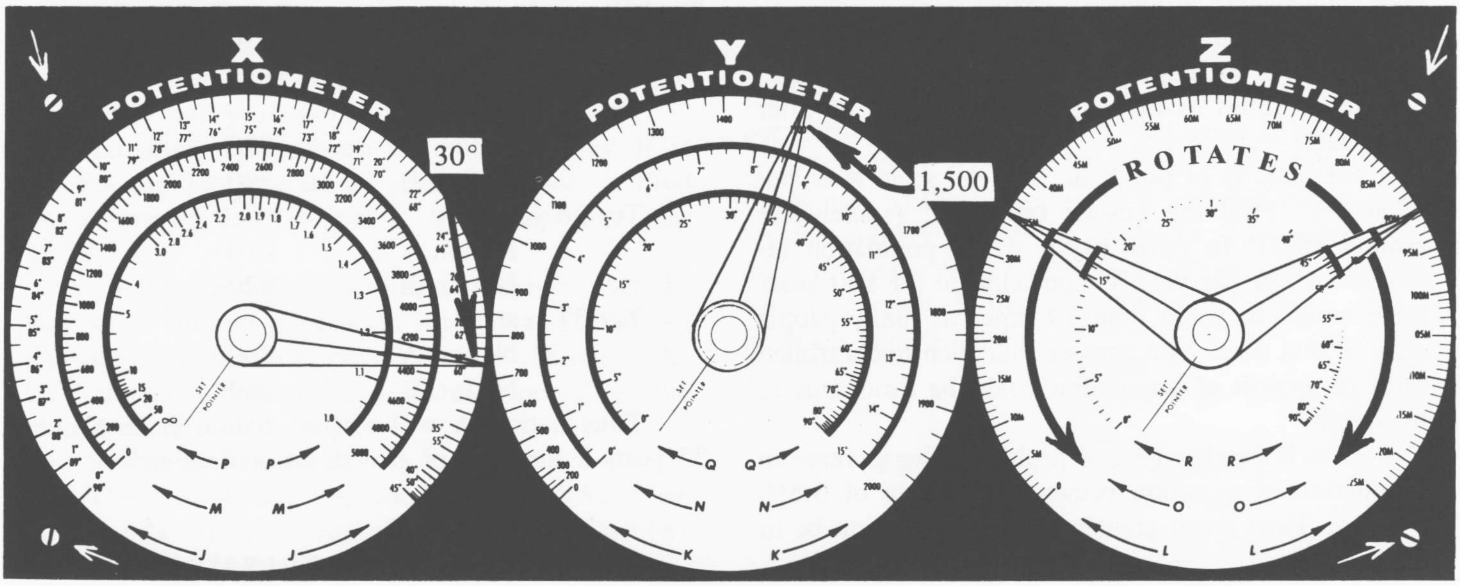


Fig. 28

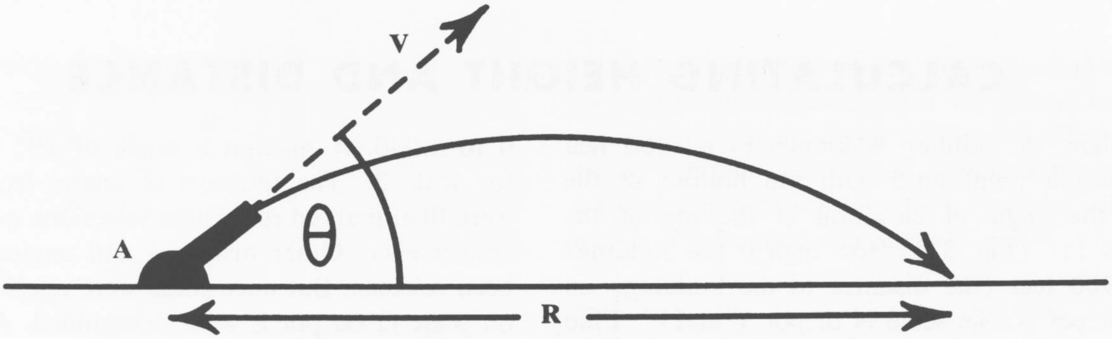


Fig. 29

attached to the computer. However, it should be possible to solve the problems with the computer even if the explanations are not fully understood.)

The formula for the range of a projectile is:

$$R = \frac{V^2}{32} \sin 2\theta$$

(If you are interested in the derivation of this formula, you will find it in some college physics textbooks). We may rewrite this:

$$32R = V^2 \sin 2\theta$$

Consider V^2 as one variable quantity and $\sin 2\theta$ as another. The above expression then resembles the form $XY = Z$ and may therefore be solved by your computer provided that the scales are marked off properly.

The horizontal position of a gun occurs when the gun is at an angle of 0° to level ground. A vertically positioned gun is at an angle of 90°. The sines of angles start at a value of 0 for 0° and go up to a value of 1 for 90°. On your computer, the values of the sines of all angles from 0° to 90° were marked off around the circle on scale J.

Now note that the formula refers to an angle of 2θ. This means that when the gun is inclined at an angle (θ) of 45°, the angle 2θ will be 90°. Therefore, we mark the maximum angle at the end of the scale 45°, not 90°. It is at this angle that the maximum range is achieved.

Note that scale J is not linear (evenly spaced) but that the spacings for the angles decrease as you go around toward larger angles up to 45°. Note also that alongside each angle is its complement. This occurs because if you double an angle (2θ) the value of its sine is the same as that of double its complement.

It was decided to set the maximum value of the velocity at 2,000 feet per second, as a reasonable figure for projectile velocities. The scale was then marked off according to V^2, not V. Thus scale K of pot Y shows spacings that are not linear, but that increase rapidly.

When the pointers for X and Y are set at the largest values (extreme right side of the scales), we should get the largest value for Z. A calculation was therefore made of the range for $\theta = 45°$ (the maximum) and the maximum velocity of 2,000 feet per second. This range amounts to 125,000 feet (125M). This figure was then set at the extreme end of dial L of pot Z. The numbers for scale L were then subdivided evenly (linearly). The actual method of subdividing numbers for the scale is shown in a later section of the book.

TRY THESE PROBLEMS:

(Answers at back of book)

25) Find the range of a projectile with a muzzle velocity of 1,600 feet per second when the cannon is elevated at an angle of 35° from the ground.

26) Find the two angles at which a gun must be pointed to strike an object 30,000 feet away, if the muzzle velocity is 1,200 feet per second.

27) What is the muzzle velocity of a gun that has a range of 11,000 feet when the angle is 37° from the ground?

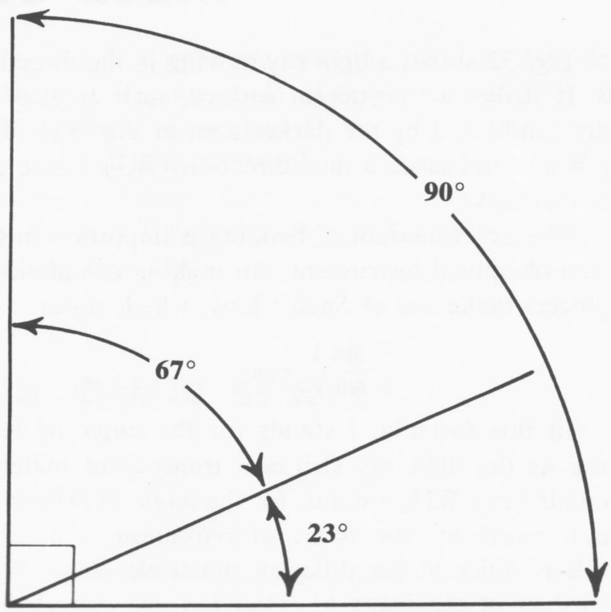


Fig. 30

CALCULATING HEIGHT AND DISTANCE

Problem: A building is known to be 900 feet away. From a point level with the bottom of the building, the angle of elevation of the top of the building is 11° (Fig. 31). How high is the building?

Set 900 feet (the distance to the building) on scale M of pot X. Set scale N of pot Y at 11°. Find the answer in the usual way on scale O of pot Z. It is approximately 175 feet.

Why does it work?

Scales M, N, and O are based upon the relationship:

$$\tan A = \frac{a}{b}$$

In Fig. 31, a triangle is shown in which *a* is the height of a distant object on level ground, *b* is the distance to the object, and A is the angle of elevation of the top of the distant object. The above formula then applies. It may be rewritten in the form:

$$a = b \tan A$$

or **Height of Object = Distance to Object × tan A**

Note that this is of the basic form $XY = Z$, in which X is the distance to the object, Y is tan A (the tangent of the angle of elevation to the top of the object), and Z is the height of the object.

A maximum distance of 5,000 feet was selected for scale M, which was then marked off linearly from 0 to 5,000. A maximum angle of 15° was selected for scale N. The tangents of angles from 0 to 15° were then marked off. These selections were made for convenience. Other distances and angles might have been selected. But once these were chosen, the values on scale O on pot Z were determined. A calculation showed that a distance of 5,000 feet and an angle of 15° corresponded to a height of 1,340 feet. This distance was therefore set at the maximum point of scale O on pot Z. This scale was then subdivided into equal divisions (linearly).

TRY THESE PROBLEMS:

(Answers at back of book)

28) A steeple is 3,200 feet distant. The angle of elevation of the top of the tower is 6°. What is the height of the tower?

29) The top of a bridge across a river is known to be 270 feet high. If the angle of elevation to the top of the bridge is 9°, when observed from the water's edge, how far away is the bridge?

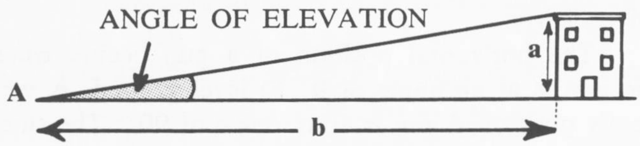


Fig. 31

INDEX OF REFRACTION

Fig. 32 shows a light ray moving in the direction AB. It strikes a transparent surface, such as glass or water (indicated by the darker area in Fig. 32). The ray bends and takes a new direction (BC) inside the new material.

The exact amount of bending is important in the design of optical instruments. In making calculations, engineers make use of Snell's Law, which states:

$$\frac{\sin i}{\sin r} = n$$

In this formula, *i* stands for the *angle of incidence* as the light ray strikes a transparent material from air (Fig. 32), *r* stands for the *angle of refraction* and *n* represents the *index of refraction,* a number which is different for different materials. Some typical values of the *index of refraction* are indicated in Table I.

TABLE I
INDEX OF REFRACTION

Vacuum	1.00
Air	1.0003
Water	1.33
Ether	1.35
Alcohol (ethyl)	1.36
Alcohol (amyl)	1.41
Hydrogen peroxide	1.41
Olive oil	1.48
Glass (crown)	1.51
Rock salt	1.54
Quartz	1.54
Turpentine	1.57
Glass (dense flint)	1.71
Diamond	2.47

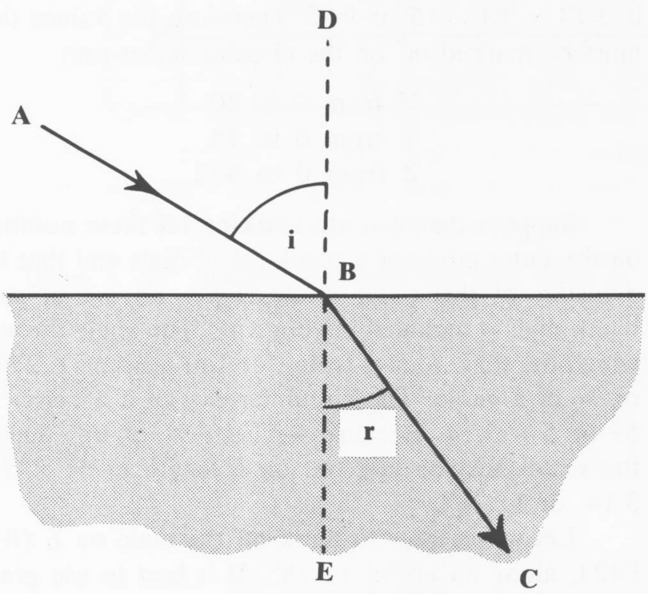


Fig. 32

There is an interesting fact about this formula which affected the design of the scales on your computer. Values of the index of refraction ordinarily start at 1 and go up to about 2.5. The region of numbers from 0 to 1 was, therefore, eliminated from your computer. Scale P is based on the following modification of Snell's Law:

$$\frac{1}{n} \cdot \sin i = \sin r$$

This now resembles the form $XY = Z$, provided that $\frac{1}{n}$ is marked off on the scale for pot X and the sines of angles are marked off on the scales for pots Y and Z. Note the manner in which the numbers for the index of refraction appear on scale P. At the zero position of the pointer, note the symbol ∞, representing *infinity*. The numbers decrease very rapidly as you go around the scale to the right and finally end up at 1.

As a practical matter, numbers higher than 2.5 are not needed because most substances have indexes of refraction from about 1.3 to 2.5. However, additional numbers have been included because it may be of interest to see what would happen if other indexes of refraction did occur.

One way scientists identify a material is by measuring the index of refraction and comparing it with the values recorded in tables similar to Table I.

Note that the index of refraction for air is very close to that of a vacuum. It is sufficiently accurate for most purposes to assume that a light ray coming from air bends as if coming from a vacuum, and so we can use 1.00 in our calculations.

Problem: a light ray enters water (from air) at an angle of incidence of 62°. What is its new direction in the water?

Set the index of refraction of water (1.33) on scale P of pot X. Set the angle of incidence (62°) on scale Q of pot Y. Find the answer (angle of refraction) on scale R of pot Z in the usual way. The answer is about 42°.

In general terms, the index of refraction appears on scale P of pot X. The angle of incidence appears on scale Q of pot Y. The angle of refraction appears on scale R of pot Z.

TRY THESE PROBLEMS:

(Answers at back of book)

30) What is the index of refraction of a crystal if the angle of incidence of a ray of light entering the crystal is 68° while its angle of refraction is 43°?

31) If a light ray traveling from air into dense flint glass has an angle of refraction of 33°, what was its angle of incidence?

32) If the angle of incidence of a light ray striking a material from air is 75°, what is its angle of refraction for diamond? For dense flint glass? For crown glass? For water?

HOW TO MAKE YOUR OWN SCALES

Fig. 33 shows an oval-shaped figure called an *ellipse*. You might think of it as a kind of flattened circle. The area of such an ellipse is given by the formula $A = \pi ab$ where a is the long dimension (as shown in Fig. 33), b is the short dimension, and π is the number 3.14. . . . The three *variables* in this case are A, a, and b.

Put this into the form $ab = \frac{A}{\pi}$.

Note that this is one type of linear function

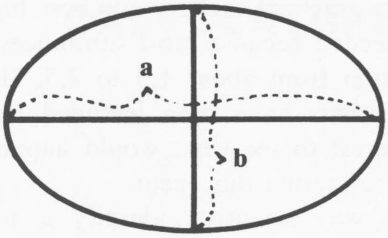


Fig. 33

that greatly resembles $XY = Z$. If the formula were $ab = A$, it would be possible to use the outer scales (A, B, C) of pots X, Y, and Z just as they are. However, the fact that A is divided by π requires that one of the three scales be modified to produce correct answers.

Furthermore, we may wish to use scales with numbers other than 10, 100 and 1,000, originally used for X, Y, and Z. How would such a new set of scales be made?

First, decide on a set of values for a and b. Except in extreme cases a and b will usually be close to each other. Suppose that you set a maximum for a (pot X) of 20 units and a maximum for b (pot Y) of 15 units.

Since $A = \pi ab$, the maximum value on pot Z is $3.14 \times 20 \times 15$ or 942. Therefore the values that must be marked off on the circular scales are:

X from 0 to 20
Y from 0 to 15
Z from 0 to 942

Suppose that you are marking off these numbers on the outer circle of a blank set of dials and that the diameter of this outer circle is 5″. (A set of such blank dials is included in your kit.) The angle through which the shaft rotates from 0 to full position is 270°, or ¾ of a circle. The circumference of a 5″ circle is 5π or 5×3.14. Because we need ¾ of this length, the scale must be laid out on a length of $¾ \times 5 \times 3.14$, or 11.78″.

Let us see how to mark off the scale on Z (0 to 942), along an arc of 11.78″. It is best to use graph paper for this purpose. Tape several sheets of ordinary graph paper together to make one large sheet about $20'' \times 15''$, or larger. Be sure that the lines are straight when you fasten the sheets of graph paper together.

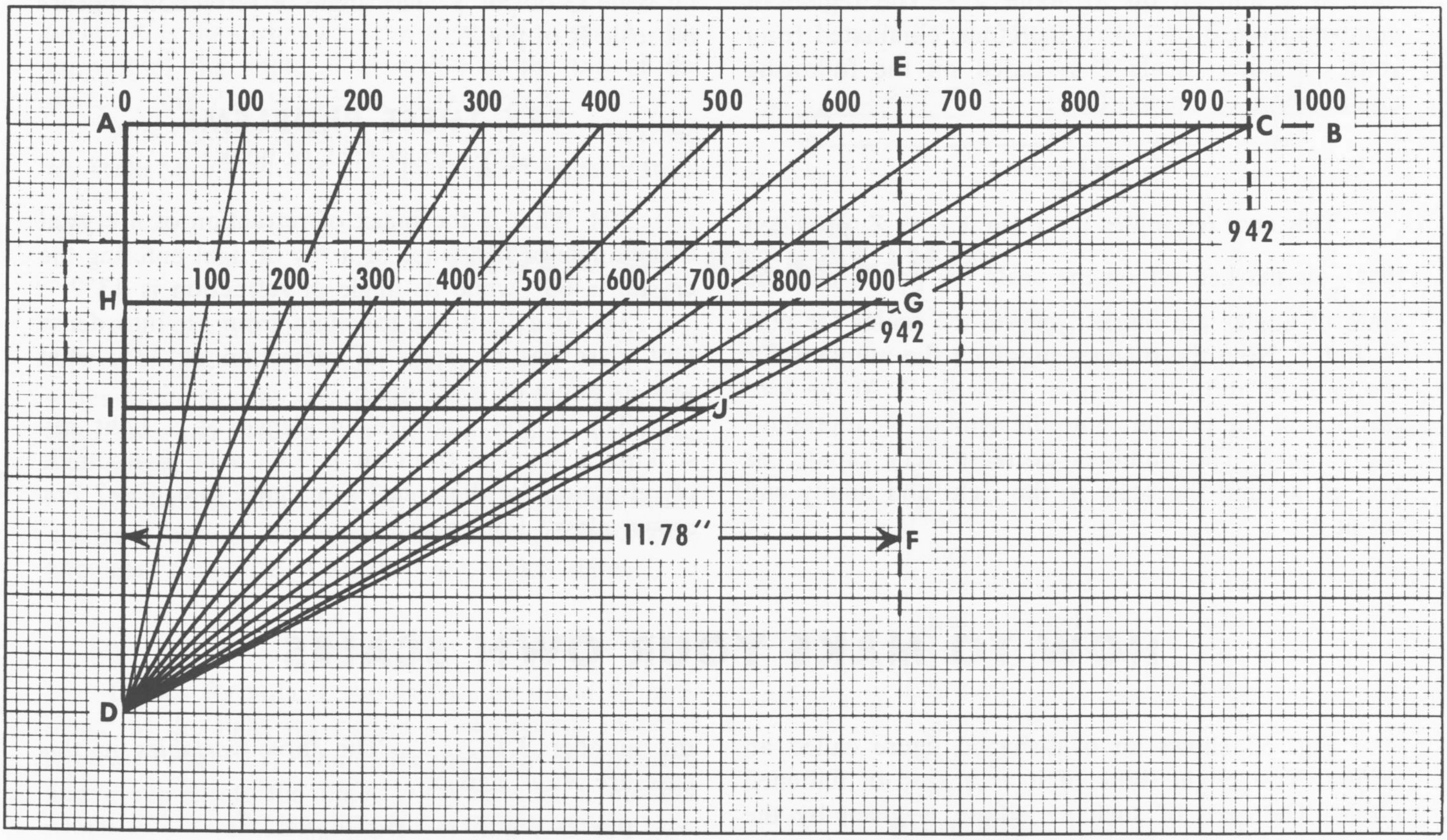


Fig. 34

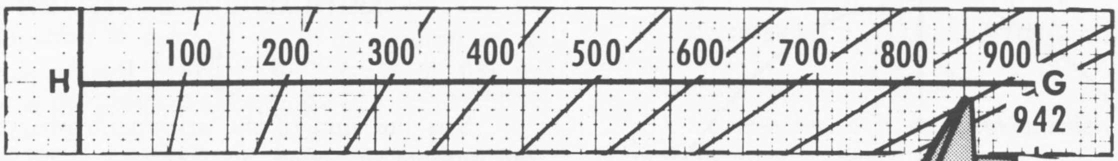
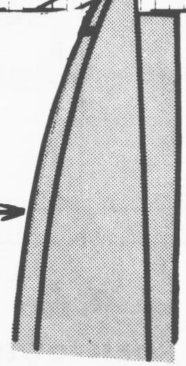


Fig. 35



Draw a 20″ straight line near the long edge of the sheet (AB in Fig. 34). Let the end of this line represent 1,000. Mark off equal divisions along the graph paper to produce divisions of 100, 200, 300, etc., up to 1,000. Locate 942 on this line as closely as you can (point C).

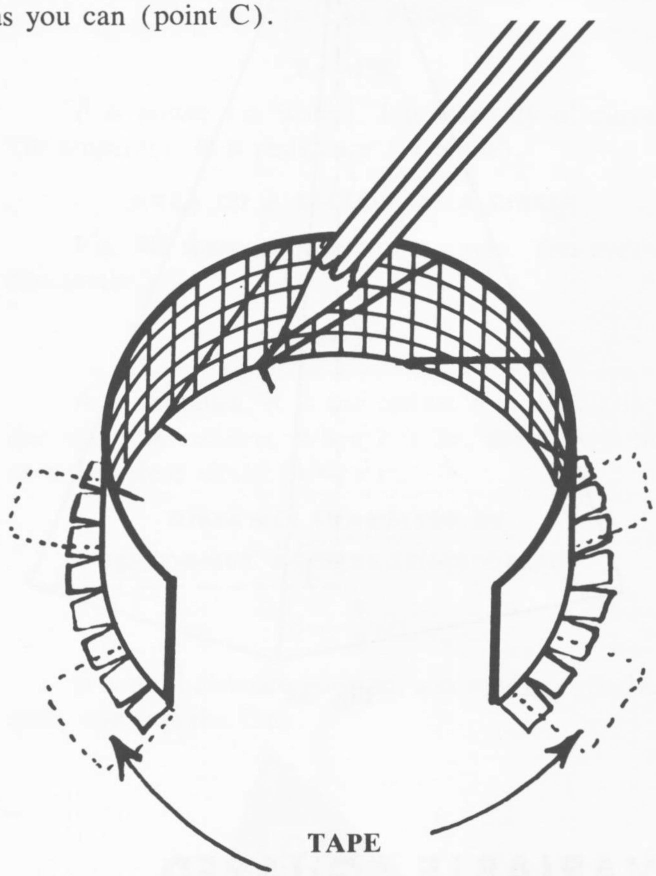


Fig. 36

Draw AD at right angles to AB. Locate D anywhere on the line, as far from A as possible. Draw DC. Then draw lines from D to the 100 point on AB, the 200 point, etc., up to the 942 point (C).

Draw line EF parallel to AD and 11.78″ from it. This line intersects CD in point G. Draw GH parallel to AB. The length of the line GH now equals the required length of the scale (11.78″) and is marked off in equal divisions from 0 to 942.

Now cut out a strip of paper on both sides of GH (dotted area in Fig. 34). Fold along line GH. Make short vertical cuts with scissors as shown at B in Fig. 35. You will then be able to curve line GH around the arc of the 5″ diameter circle, as shown in Fig. 36. Use a bit of tape to hold the paper in place as you mark off the divisions. You can then mark off as many smaller divisions as needed. This may be done by eye, or more accurately with a compass or graph paper.

Use a similar procedure for marking off the X and Y scales. Or you can make a tracing of one of the printed scales to obtain the evenly spaced divisions and then assign proper values, as you require.

If your scale has a diameter smaller than 5″, reduce the final length to equal the smaller arc length. Such a shortened length is shown by line IJ in Fig. 34.

LINEAR FUNCTIONS

The following formulas are of the form:

$$Z = CXY$$

C is some constant (fixed number), and X, Y, and Z are "variables" that may be assigned any value you wish.

AREA OF A PARABOLA

$$A = 2\,lh$$

A is area, l is "length," and h is "height," as shown in Fig. 37.

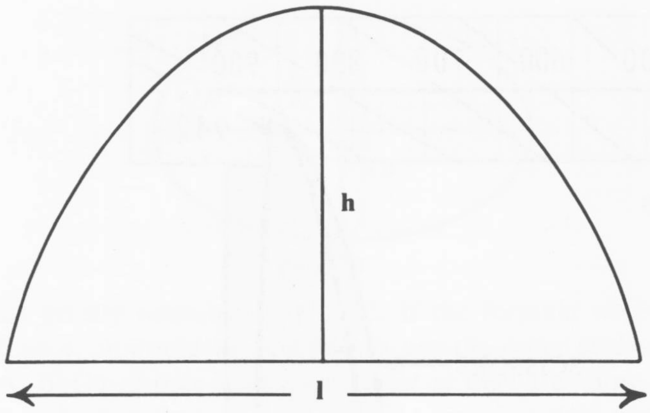


Fig. 37

HORSEPOWER

$$H = \frac{W}{550T}$$

H is horsepower, W is the work done in foot-pounds, and T is the time in seconds.

VOLUME OF PYRAMID

$$V = \tfrac{1}{3}Ah$$

V is volume, A is area of base, and h is the altitude (Fig. 38). The base of the pyramid may be of any shape.

INDUCTIVE REACTANCE

$$X_L = 2\pi fL$$

X_L is the inductive reactance (in ohms), f is the frequency of the alternating current, and L is the inductance in henries.

ARITHMETIC PROGRESSION

$$S = \frac{n}{2}(a + l)$$

S is the sum of the arithmetic progression, n is the number of terms, a is the first term, and l is the last term. Consider (a + l) as one variable. When solving with the computer, add a and l and use the resulting number on either pot X or pot Y.

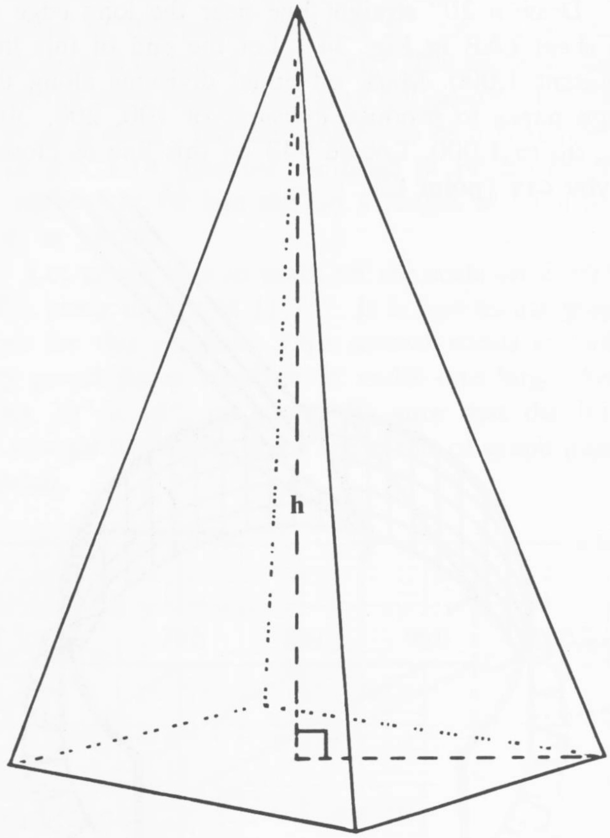


Fig. 38

FORMULAS WITH ONE VARIABLE SQUARED

The formulas in this section are of the type in which two of the variables are linear and the third is squared. These may be subdivided still further into the two types:

$$Z = C\,X\,Y^2 \text{ and } Z^2 = C\,X\,Y$$

As before, C is a constant number, different for each formula.

To mark off the scales for these formulas, two

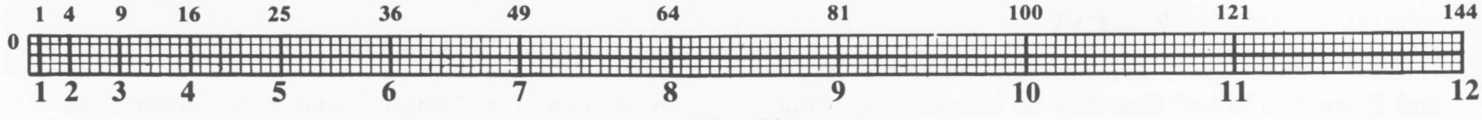


Fig. 39

are made linear (evenly divided markings) and the squared function is marked off as a squared set of numbers, as shown in Fig. 39. (Scale K for the velocity of a projectile is of this form.)

ILLUMINATION CAUSED BY A SOURCE OF LIGHT

$$I = \frac{C}{D^2}$$

I is illumination (in foot-candles), C is candle-power of light source, and D is distance from the light source in feet.

Put this formula in the form:

$$C = ID^2$$

ELECTRICAL POWER

$$P = I^2R$$

P is power (in watts). I is intensity of current (in amperes). R is resistance (in ohms).

AREA OF A SECTOR OF A CIRCLE

Fig. 40 shows a sector of a circle. The area of this sector is:

$$A = \frac{1}{2}r^2\,\theta$$

A is the area, R is the radius, θ is the angle of the sector in *radians.* When θ is 2π, the area is that of a complete circle: $A = \pi\, r^2$.

DISTANCE TRAVELLED BY UNIFORMLY ACCELERATING OBJECT

$$d = \frac{1}{2}at^2$$

d is total distance covered, a is uniform acceleration, and t is the time.

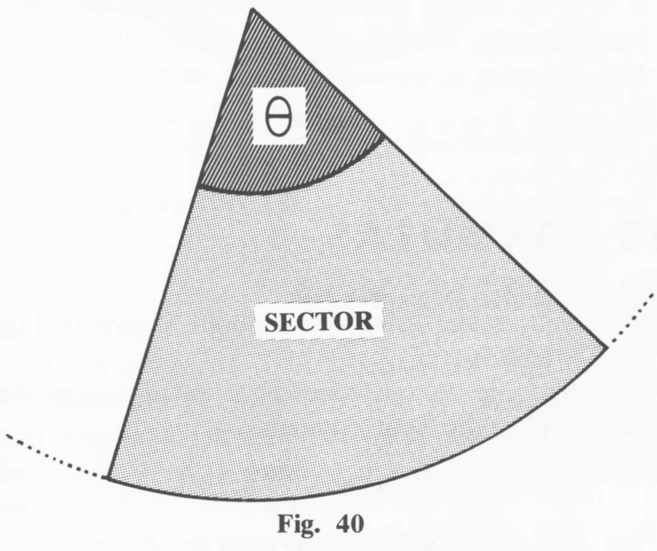


Fig. 40

FINAL VELOCITY OF UNIFORMLY ACCELERATING OBJECT

$$V^2 = 2ad$$

V is velocity, a is uniform acceleration, and d is total distance covered.

CIRCULAR MOTION

$$a = \frac{v^2}{r}$$

a is the acceleration of an object moving in a circle, v is the velocity along its path, and r is the radius (distance to the center).

Put this in the form $v^2 = ar$.

KINETIC ENERGY

$$K = \frac{1}{2}mV^2$$

K is kinetic energy, m is mass, and V is velocity.

VOLUME OF A CYLINDER

$$V = \pi r^2 h$$

V is volume, r is the radius of the base, and h is the altitude.

VOLUME OF AN OBLATE SPHEROID

An oblate spheroid is a flattened sphere. The earth has a similar shape.

$$V = \frac{3}{4}\pi a^2 b$$

The letter "a" is the radius of the wide part ("equator"), and b is the radius of the narrow part (from the "North Pole" to the "South Pole").

VOLUME OF A CONE

$$V = \frac{1}{3}\pi r^2 h$$

V is the volume, r is the radius of the circular base, and h is the altitude.

VOLUME OF A SPHERICAL SECTOR

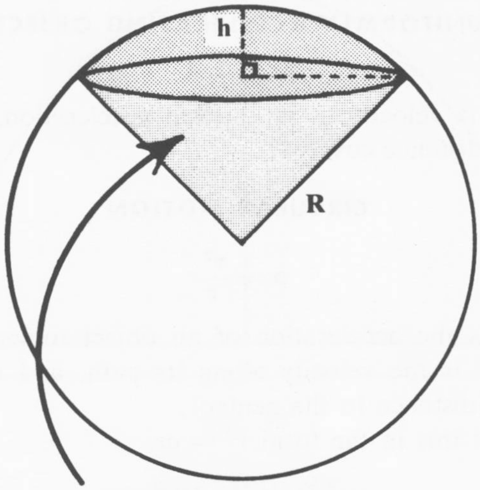
$$V = \frac{2}{3}\pi r^2 h$$

See Fig. 41.

VOLUME OF A TORUS

A torus is a doughnut shape (Fig. 42). Its volume is given by the formula:

$$V = 2\pi^2 Rr^2$$



SPHERICAL SECTOR

Fig. 41

V is volume, and R and r are the radii indicated in Fig. 42.

AREA OF A TORUS

$$A = 4\pi^2 Rr$$

See Fig. 42.

VELOCITY OF A MOLECULE OF A GAS

$$MV^2 = 3RT$$

M is the molecular weight, V is the "root mean square" velocity (a kind of average velocity), and T is the Kelvin (absolute) temperature. When the velocity is in centimeters per second, the constant R is 8.314×10^7.

It might be of interest to change the units in the above formula to miles per second and the temperature to Celsius (centigrade) degrees on your scale. It would then be possible to use the computer rapidly to find the average velocity of molecules of any gas at any temperature.

VELOCITY OF A MOLECULE OF A GAS

Another interesting formula calculates the (average) velocity of a molecule of a gas if the density of the gas and pressure are known.

$$V^2 = \frac{3P}{d}$$

or

$$3P = dV^2$$

In this formula the pressure is in dynes per square centimeters. The velocity and density are in units of centimeters, grams, and seconds.

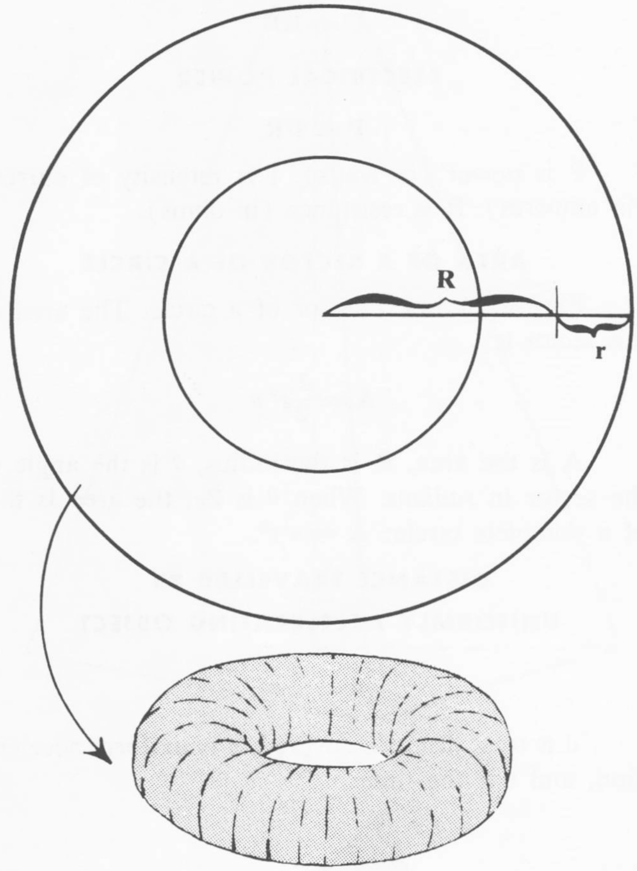


Fig. 42

TRIGONOMETRIC FORMULAS

The following trigonometric functions contain three variables of the form in which one quantity multiplied by a second equals a third, and they may therefore be solved with your computer.

To make appropriate scales, it will be necessary to mark off lengths in proportion to the trigonometric functions taken from a table and then shrink them to fit your scale, using the methods shown in Figs. 34, 35, and 36.

Here are two trigonometric relationships that may be put onto your computer.

$$\tan a = \frac{\sin a}{\cos a}$$

$$\sin 2a = 2 \sin a \cos a$$

PERIMETER OF A REGULAR POLYGON

Some mathematical functions involve combinations of three variables in complex ways. Consider the following:

Fig. 43 shows a regular polygon, which has all sides and angles equal. The perimeter (P) of such a regular polygon is given by the formula:

$$P = 2nR \sin \frac{\pi}{n}$$

R is the radius of the circumscribed circle (the distance from center to one corner), and n is the number of sides.

Note that there are three variables (P, n and R) and that they are all connected by multiplication or division in some manner. Recombine the variables as follows:

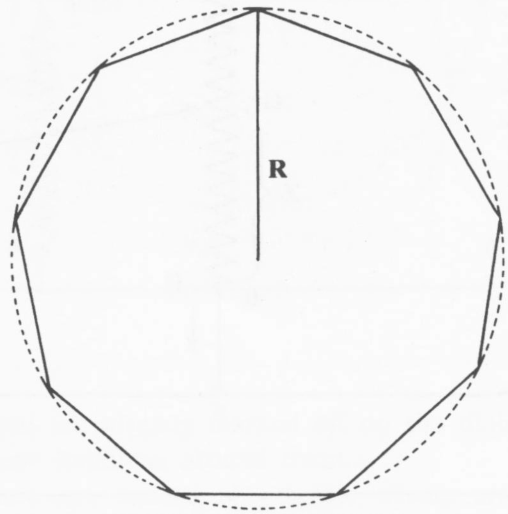
$$P = 2\,R\,(n \sin \frac{\pi}{n})$$

This is now resembles the general form $Z = CXY$ and can be put onto scales for your computer. However, it will be necessary to mark off one of the scales (either X or Y) in proportion to $(n \sin\frac{\pi}{n})$. Calculate $(n \sin\frac{\pi}{n})$ for a number of values of n, up to about 15. Then mark off the scales by the method used in Figs. 34, 35, and 36.

AREA OF REGULAR POLYGON

$$A = \frac{1}{2}R^2\,(n \sin \frac{2\pi}{n})$$

See Fig. 43 and the previous problem.



REGULAR POLYGON
Fig. 43

RADIOACTIVE DECAY

Radioactive substances "decay" or change into other materials at a fixed rate. A fixed percentage remains at the end of a given period. For example, suppose that 10% of a certain substance decays after a day. The decay constant is said to be .10 (per day). Start with 1 pound of material. At the end of one day the amount left would be .9 pound. At the end of the second day the amount left would be $.9 \times .9$ or .81. At the end of the third day the amount would be $.9 \times .9 \times .9$ or .73. The formula in general would be

$$A = (1 - d)^t$$

A represents the amount left after a number of periods of time (t), assuming the original amount is 1, and d represents the radioactive decay constant. This greatly resembles the formula for compound interest and growth, $A = (1 + i)^n$, except that decay and reduction occur instead of growth.

The radioactive decay function may be put onto your computer by changing to logarithms.

$$A = (1 - d)^t$$

$$\log A = t \log (1 - d)$$

This resembles the form $XY = Z$. Let the scale on X represent t, the scale on Y represent log $(1 - d)$ and the scale on Z represent log A.

ADVANCED SCIENCE PROJECTS

There are many ways in which the three potentiometers could be wired up. Every variation alters the manner in which the positions of pots X, Y, and Z affect each other to produce a zero reading on the meter.

For example, suppose that the three pots are connected as shown in Fig. 44.

Let **X** = the decimal portion of resistance indicated by the rotating shaft of the 10 ohm pot.

Let **Y** = the same for the 100 ohm pot.

Let **Z** = the same for the 1000 ohm pot.

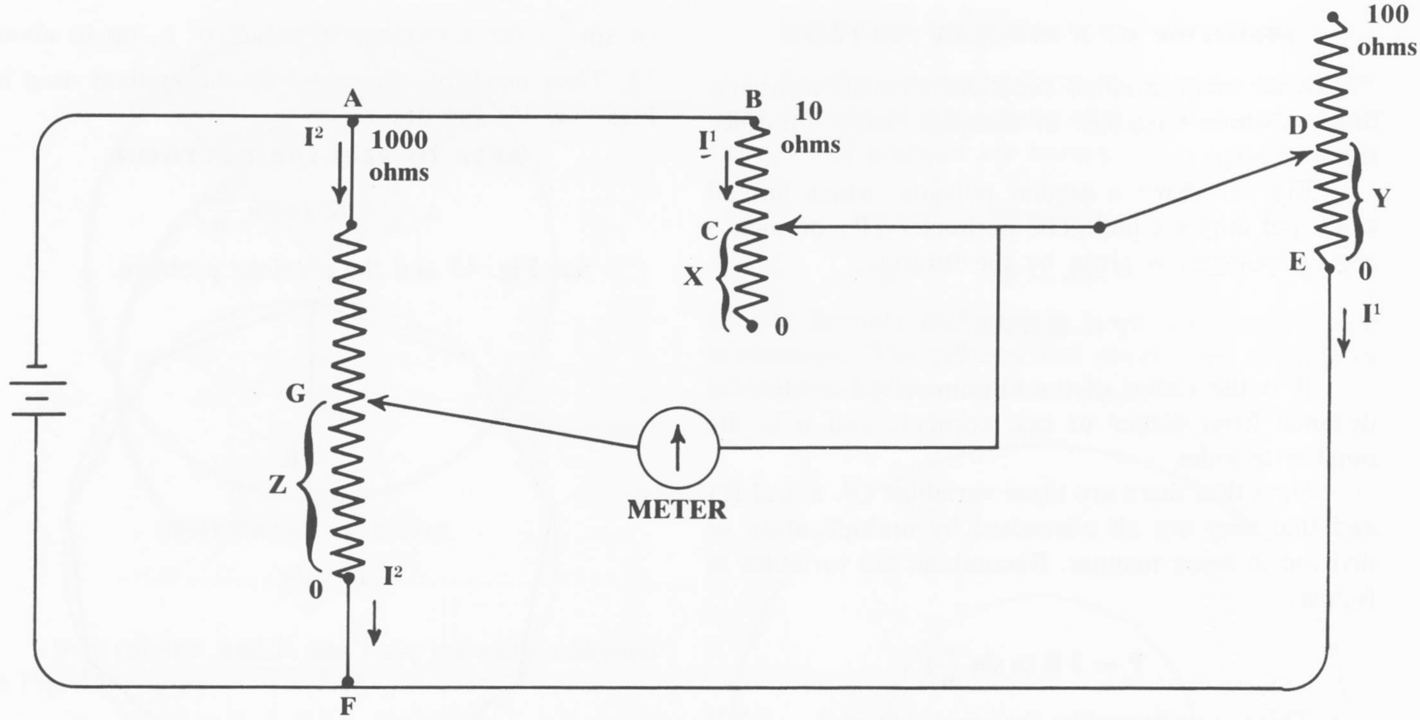


Fig. 44

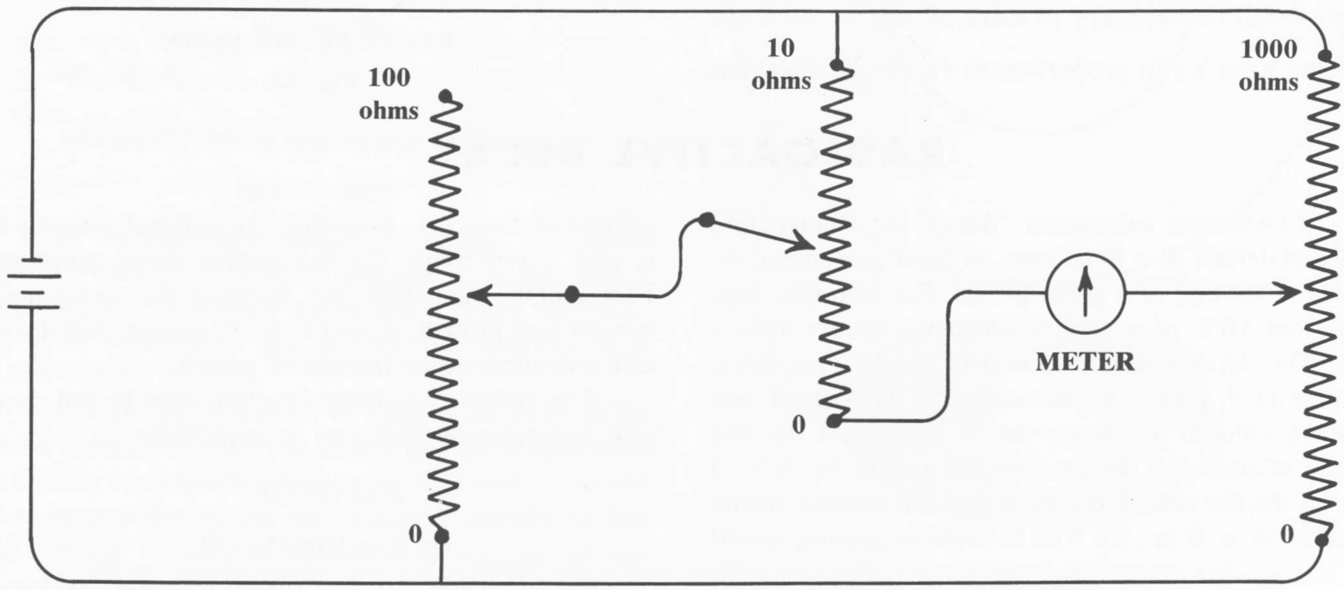


Fig. 45

Points A and B are at the same potential. Points F and E are at the same potential. Therefore the voltage drop from A to F must equal the voltage drop from B to E (going through points B, C, D, E). We now make use of Ohm's Law, which states that E = IR, where E is voltage or potential drop (P.D.), I is current (in amperes), and R is resistance (in ohms). Assume no current in the meter.

Let I_1 = current through BCDE.

Let I_2 = current through AF.

Resistance from B to C is $10(1 - X)$ ohms.

Resistance from D to E is 100Y ohms.

Total resistance from B to E is:

$$10(1 - X) + 100Y.$$

P.D. from B to E is $I_1\,[10(1 - X) + 100Y]$.

P.D. from A to F is $1000I_2$.

Since the P.D. from B to E is the same as from A to F, then:

1) $I_1[10(1 - X) + 100Y] = 1000I_2$

Now if the meter at M reads zero the potential

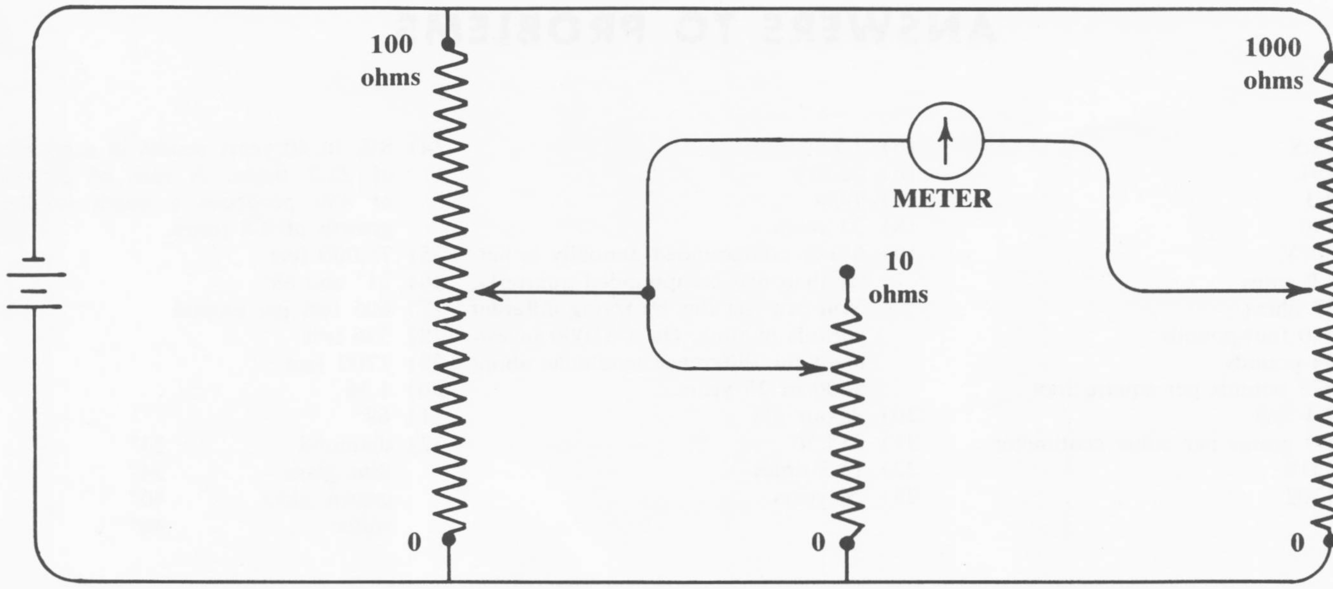


Fig. 46

drop from G to F must equal that from D to E.

P.D. from G to F is $1000ZI_2$.

P.D. from D to E is $100YI_1$.

Therefore:

2) $100YI_1 = 1000ZI_2$

Divide equation 1 by equation 2. We obtain:

$$3)\quad \frac{I_1\,[10\,(1-X)+100Y]}{100YI_1} = \frac{1000I_2}{1000ZI_2}$$

This simplifies to:

$$4)\quad \frac{1-X+10Y}{10Y} = \frac{1}{Z}$$

$$5)\quad Z = \frac{10Y}{1-X+10Y}$$

If each of the three scales is marked off as decimal fractions from 0 to 1, then substituting any values for two of the three variables and rotating the shaft for the third so as to give a zero reading on the meter will automatically produce a correct solution for the third variable in equation 5 above.

A wide variety of functions of similar form can be set up for the circuit, using different numbers on each dial. Logarithms or trigonometric scales widen the possibilities still further.

There are many different ways in which the three pots and the meters can be connected together. Figs. 45 and 46 show two additional circuits. Can you analyze each and set up functions on pots X, Y, and Z to solve problems? To save time you might use scales that are already marked off on the dials and build your functions around them.

There are dozens of possible wiring arrangements. Even a reversal of the 0 and full scale terminals of a pot alters the nature of the function. An analysis of the types of functions that may be solved would make an excellent science project.

You can combine the pots and meter with sets of rotating switches that would permit you to switch in different connections and thereby quickly obtain a wide variety of functions.

You can combine these circuits with different scales to obtain complex combinations of trigonometric, logarithmic, and other functions.

You can add fixed resistors in different ways to increase the possibilities still more.

With this kit, you have obtained a glimpse of the possibilities of using analog computers to solve problems. Giant analog computers make use of a wide range of electrical and mechanical means to produce an amazing variety of solutions to functions. Capacitors, resistors, inductances, transistors, vacuum tube circuits, special motors, and gearing systems can be put together in an infinite number of combinations.

The use of analog computers began only two decades ago. There is no doubt that intensive development in this field will occur in the next two decades. Perhaps you will participate in creating new machines and applications by working in this new field yourself.

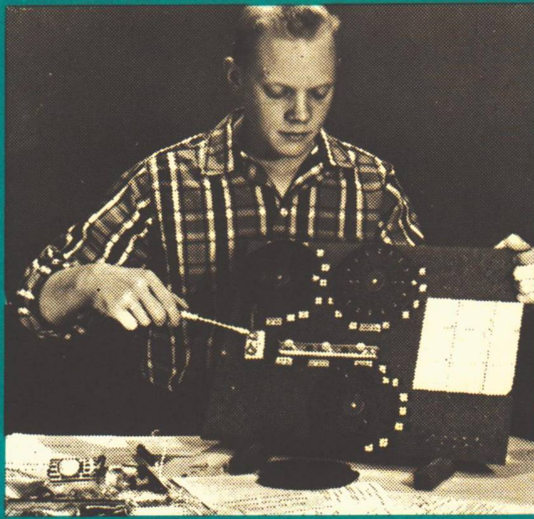
ANSWERS TO PROBLEMS

1) 23.5
2) 791
3) 9.3
4) 490
5) 307V
6) 7.9 amps
7) 57 ohms
8) 640 foot-pounds
9) 34 pounds
10) 575 pounds per square foot
11) 7.4 feet
12) 6.9 grams per cubic centimeter
13) 1310
14) 1702
15) 7.5
16) 22,810
17) 7454
18) 23 years
19) 6½% compounded annually is better than 6% compounded quarterly. You can test this by trying different periods of time. On a $1000 investment the difference would be about $400 in 25 years.
20) About 6%
21) $15.50
22) 11.5 times
23) 24 years
24) 8% in 40 years results in a growth of 21.7 times. A rate of growth of 4% produces a much smaller growth of 4.8 times.
25) 75,000 feet
26) 21° and 69°
27) 605 feet per second
28) 336 feet
29) 1700 feet
30) 1.36
31) 69°
32) diamond .... 23°
flint glass .... 34°
crown glass ..... 40°
water .... 46°

Other Interesting Science Materials

BRAINIAC® K-30

New, improved version of the famous *computer circuits kit*



Six rotating multiple switch disks make it possible to connect together an almost limitless number of switching arrangements. The detailed instructions explain how to arrange 50 different circuits, play games such as tit-tat-toe, solve puzzles, demonstrate logic and reasoning problems, operate electric quiz machines, code and decode, and show principles of combination locks. The circuits are assembled with nuts and bolts and operate on one flashlight cell. Patented "wipers" provide smooth yet firm contact when the switches are rotated.

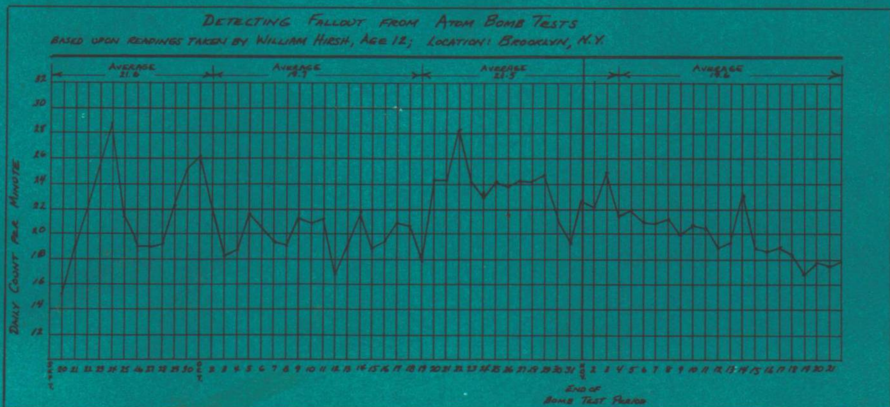
A basic electrical kit for young people age 12 up, and for adults.

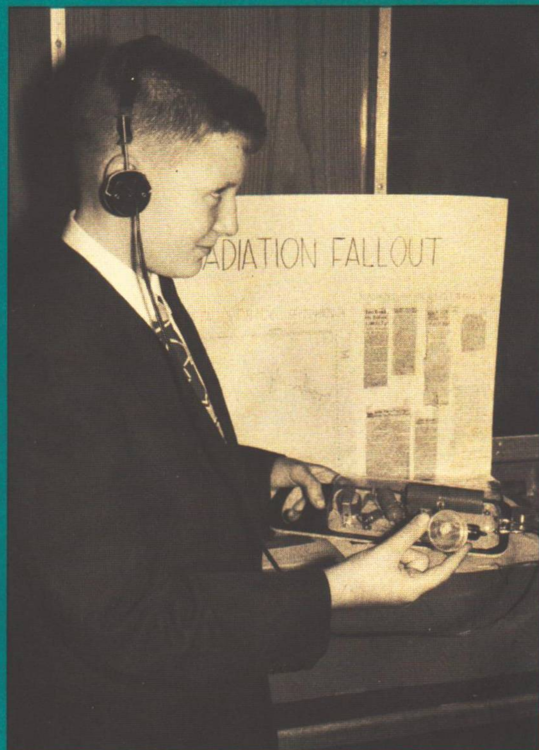
#1119 $16.95

RADIATION DETECTION KIT

William Hirsh won first-prize in the science fair for Brooklyn, New York with his report on Radiation Fallout from Atom Bomb Tests, in which he used the Science Materials Center Radiation Detection Kit, featuring a Geiger Counter which is easily assembled without any soldering. Over 40 experiments are detailed in the easy-to-read, 48-page instruction book.

#0105 $18.50





PHYSICS OF LIGHT KITS

LIGHT POLARIZER KIT

A 28-page manual of experiments clearly explains the nature of polarized light, how to use the polarizers in the kit and details dozens of experiments and science projects that may be performed.

#0102 $2.25

COLOR SPECTROSCOPE KIT

An assembled hand spectroscope utilizes a diffraction grating with 13,400 lines per inch to form spectra of various sources. Bright line, dark line and continuous spectra may be observed. A 16-page manual explains the wave theory of light and diffraction and shows how to measure the wave-length.

#0100 $2.50